أمثلة عامة على التكاملات

① $\int \frac{e^x}{e^x+1}dx$

$y=e^x \to dy=e^x dx \to dx=\frac{dy}{e^x}$

$\int \underbrace{\frac{y}{y+1}\times\frac{dy}{e^x}}_{\text{كسور}}=\int \frac{y}{(y+1)(y)}dy$

$=\ln|y+1|+c=ln|e^x+1|+c$

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② $\int \frac{1-\sin x}{x+\cos x}dx$

$=\ln|x+cos\,x|+c$

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③ $\int \frac{1-\sin^3 x}{1-\sin x}dx$

$=\int \frac{\cancel{(1-\sin x)}(1+\sin x+\sin^2 x)}{\cancel{1-\sin x}}dx$

$=x-\cos x+\int \frac{1}{2}(1-cos\,2x)\,dx$

$=x-cos\,x+\frac{1}{2}\left(x-\frac{sin\,2x}{2}\right)+c$

$=x-\cos x+\frac{1}{2}x-\frac{1}{4}\sin 2x+c$

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④ $\int \frac{dx}{sin^2\,x\,cos^2\,x}$

$=\int \frac{dx}{\left(\frac{\sin^2 x}{2}\right)^2}=\int \frac{4}{sin^2\,2x}dx$

$=4\int \csc^2 2x\,dx$

$=\frac{-\cancel{4}\cot 2x}{\cancel{2}}+c=-2\cot 2x+c$

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⑤ $\int \tan x \ln\cos x\,dx$

$y=\ln\cos x \to dy=\frac{-\sin x}{\cos x}dx$

$\to dy=-\tan x\,dx \to dx=\frac{dy}{-\tan x}$

$\int \cancel{\tan x}\times y\frac{dy}{-\cancel{\tan x}}$

$=\frac{-y^2}{2}+c=\frac{-(\ln\cos x)^2}{2}+c$

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⑥ $\int \frac{1}{\sqrt[3]{x}-\sqrt{x}}dx$

$x=y^6 \to dx=6y^5dy$

$\int \frac{1}{y^2-y^3}\times 6y^5\,dy$

$=\int \frac{6y^{\cancel{5}\,3}}{\cancel{y^2}(1-y)}\,dy$

$=\int -6y^2-6y-6+\int \frac{6}{-y+1}$

$=\frac{-6y^3}{3}-\frac{6y^2}{2}-6y+6\ln|1-y|+c$

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⑦ $\int \frac{2xe^x}{(1+x)^2}\,dx$

$\int 2xe^x(1+x)^{-2}\,dx$

$z=2xe^x \to dz=2xe^x+2e^x dx$

$de=(1+x)^{-2} \to e=\frac{(1+x)^{-1}}{-1}$

$\int e.dz=z.e-\int e\,dz$

$\int \frac{-2xe^x}{1+x}+\int \frac{2e^x\cancel{(x+1)}}{\cancel{x+1}}\,dx$

$=\frac{-2xe^x}{1+x}+2e^x+c$

$$= -\frac{4}{3}\ln|y| + c = \frac{-4}{3}\ln\left|1 - x^{\frac{3}{4}}\right| + c$$

⑪ $\int \dfrac{6\cos x}{(2\sin x - 1)(\sin x + 1)}\, dx$

$$y = \sin x \to dx = \frac{dy}{\cos x}$$

$$\int \frac{6\cancel{\cos x}}{(2y-1)(y+1)} \times \frac{dy}{\cancel{\cos x}}$$

$$= \int \frac{6}{(2y-1)(y+1)}\, dy \left(\text{كسور جزئية}\right)$$

⑫ $\int x^{13}(x^7+1)^{10}\, dx$

$$y = x^7 + 1 \to dx = \frac{dy}{7x^6}$$

$$y - 1 = x^7$$

$$\int \cancel{x^{13}}^{x^7} y^{10} \frac{dy}{\cancel{7x^6}}$$

$$= \frac{1}{7}\int x^7 y^{10}\, dy = \frac{1}{7}\int (y-1)y^{10}\, dy$$

$$= \frac{1}{7}\int (y^{11} - y^{10})\, dy = \frac{1}{7}\left(\frac{y^{12}}{12} - \frac{y^{11}}{11}\right) + c$$

⑬ $\int \dfrac{x^5}{(x+1)^7}\, dx$

$$= \int \left(\frac{x}{x+1}\right)^5 \times \frac{1}{(x+1)^2}\, dx$$

$$y = \frac{x}{x+1} \to dy = \frac{(x+1)(1) - (x)(1)}{(x+1)^2}$$

$$\to dy = \frac{1}{(x+1)^2} \to dx = (x+1)^2\, dy$$

$$= y^5 \times \frac{1}{\cancel{(x+1)^2}} \cancel{(x+2)^2}\, dy$$

$$= \frac{y^6}{6} + c = \frac{\left(\frac{x}{x+1}\right)^2}{6} + c$$

⑧ $\int x\cot^2 x\, dx$

$$= \int x(\csc^2 x - 1)\, dx = \int x\csc^2 x - x\, dx$$

$$= \int \underbrace{x\csc^2 x\, dx}_{\text{أجزاء}} - \int x\, dx$$

$$= -x\cot x - \int \cot x\, dx - \frac{x^2}{2} + c$$

$$= -x\cot x - \ln|\sin x| - \frac{x^2}{2} + c$$

⑨ $\int \dfrac{\sqrt{x+3}}{x^2+6x+9}\, dx$

$$= \int \frac{\sqrt{x+3}}{(x+3)^3}\, dx$$

$$= \int (x+3)^{\frac{1}{2}}(x+3)^{-2}\, dx$$

$$= \int (x+3)^{\frac{-3}{2}}\, dx$$

$$= \frac{(x+3)^{\frac{-1}{2}}}{-\frac{1}{2}} + c = \frac{-2}{\sqrt{x+3}} + c$$

⑩ $\int \dfrac{1}{\sqrt[4]{x} - x}\, dx$

$$= \int \frac{1}{x^{\frac{1}{4}} - x}\, dx \left(\text{اخرج عامل مشترك } x^{\frac{1}{4}}\right)$$

$$= \int \frac{1}{x^{\frac{1}{4}}\left(1 - x^{\frac{3}{4}}\right)}\, dx$$

$$y = 1 - x^{\frac{3}{4}}$$

$$dy = \frac{-3}{4}x^{\frac{-1}{4}} \to dx = \frac{-4}{3}x^{\frac{1}{4}}\, dy$$

$$\int \frac{1}{\cancel{x^{\frac{1}{4}}} \times y} \times \frac{-4}{3}\cancel{x^{\frac{1}{4}}}\, dy$$

⑭ $\int e^{\sin x+\ln\cos x}\,dx$

$= \int e^{\sin x} \times e^{\ln\cos x}\,dx$

$= \int e^{\sin x} \times \cos x\,dx$

$y = \sin x \rightarrow dx = \dfrac{dy}{\cos x}$

$= \int e^{x} \times \cancel{\cos x}\dfrac{dy}{\cancel{\cos x}}$

$= e^{x} + c = e^{\sin x} + c$

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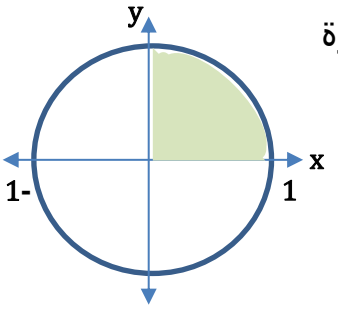
⑮ $\int_{1}^{0} \sqrt{1-x^2}\,dx$

$= -\int_{1}^{0} \sqrt{1-x^2}\,dx$

لدينا ربع مساحة الدائرة



$= -\dfrac{1}{4}\pi r^2$

$= -\dfrac{1}{4}\pi(1)^2 = -\dfrac{\pi}{4}$

معادلة الدائرة

$x^2 + y^2 = 1$

$y = \sqrt{1-x^2}$

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⑯ $\int \dfrac{1}{x\sqrt{x^2+1}}\,dx$

$y = \sqrt{x^2+1}$

$y^2 = x^2 + 1$

$2y\,dy = 2x\,dx$

$\dfrac{y}{x}dy = dx$

$\int \dfrac{1}{x \times \cancel{y}} \times \dfrac{\cancel{y}}{x}\,dy$

$= \int \dfrac{1}{x^2}\,dy$

$= \int \dfrac{1}{y^2-1}\,dy$ كسور جزئية

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⑰ $\int \dfrac{2x\sin x^2}{\sec^2 x^2}\,dx$

$= \int 2x\sin x^2 \cos x^2\,dx$

$y = x^2 \rightarrow dx = \dfrac{dy}{2x}$

$= \int \cancel{2x}\sin x\cos x\dfrac{dy}{\cancel{2x}} = \int \dfrac{\sin 2y}{2}dy$

$= \dfrac{-1}{4}\cos 2y + c = \dfrac{-1}{4}\cos 2x^2 + c$

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⑱ $\int \dfrac{1}{(x-2)^2}\sqrt{\dfrac{x+1}{x-2}}\,dx$

$y = \dfrac{x+1}{x-2} \rightarrow \dfrac{dy}{dx} = \dfrac{(x-2)(1)-(x+1)(1)}{(x-2)^2}$

$= \dfrac{x-2-x-1}{(x-2)^2} = \dfrac{-3}{(x-2)^2}$

$dx = \dfrac{(x-2)^2}{-3}dy$

$\int \dfrac{1}{\cancel{(x-2)^2}}\, y^{\frac{1}{2}}\dfrac{\cancel{(x-2)^2}}{-3}\,dy$

21. $\int \frac{\sin 2x}{\sin^2 x - 3\sin x + 2}\, dx$

$y = \sin x \to dx = \frac{dy}{\cos x}$

$\int \frac{2\sin x \cancel{\cos x}}{y^2 - 3y + 2} \times \frac{dy}{\cancel{\cos x}}$

$= \int \frac{2y}{y^2 - 3y + 2}\, dy$ كسور جزئية

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22. $\int \sin^h x \sec^{h+2} x\, dx$

$= \int \sin^h x \sec^h x \sec^2 x\, dx$

$= \int \frac{\sin^h x}{\cos^h x} \sec^2 x\, dx$

$= \int \tan^h x \sec^2 x\, dx$

$y = \tan x \to dx = \frac{dy}{\sec^2 x}$

$= \int y^h \cancel{\sec^2 x} \frac{dy}{\cancel{\sec^2 x}}$

$= \frac{y^{h+1}}{h+1} + c = \frac{\tan^{h+1} x}{h+1} + c$

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23. $\int \sqrt[3]{2x^5 - x^3}\, dx$

$= \int \sqrt[3]{x^3(2x^2 - 1)}\, dx$

$= \int x \sqrt[3]{(2x^2 - 1)}\, dx$

افرض $y = 2x^2 - 1 \to$ أكمل

$-\frac{2}{3} \times \frac{y^{\frac{3}{2}}}{3} + c = \frac{-2}{9}\sqrt{\left(\frac{x+1}{x-2}\right)^3} + c$

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⑲ $\int \frac{\sqrt{x+1}}{\sqrt{x^5}}\, dx$

$= \int \frac{\sqrt{x+1}}{\sqrt{x^4}\sqrt{x}}\, dx$

$= \int \frac{1}{x^2}\sqrt{\frac{x+1}{x}}\, dx$

$= \int \frac{1}{x^2}\sqrt{1 + \frac{1}{x}}\, dx$

افرض : $y = 1 + \frac{1}{x} \to$ أكمل

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⑳ $\int e^x \sin x\, dx$

$r = e^x \to dr = e^x\, dx$

$de = \sin x\, dx \to e = -\cos x$

$-\int e^x \sin x\, dx = -e^x \cos x + \int e^x \cos x\, dx$

$r = e^x \to dr = e^x\, dx$

$de = \cos x\, dx \to e = \sin x$

$-e^x \cos x + e^x \sin x - \int e^x \sin x\, dx$

$-e^x \cos x\, dx + e^x \sin x - \int e^x \sin x\, dx$

$\int e^x \sin x\, dx = \frac{e^x - e^x \cos x}{2} + c$

$$= \int \frac{1}{\cancel{x^2}} \sqrt{y^5}\, \frac{\cancel{x^2}}{5} dy = \frac{1}{5} \int y^{\frac{5}{2}}\, dy$$

$$= \frac{1}{5} \left(\frac{y^{\frac{7}{2}}}{\frac{7}{2}} \right) + c = \frac{2}{35} \sqrt{\left(1 - \frac{5}{x}\right)^7} + c$$

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27. $\int \frac{\sqrt[3]{x^3 - x}}{x^4}$

$$\int \frac{1}{x^3} \sqrt[3]{\frac{x^3 - x}{x^3}}\, dx = \int \frac{1}{x^3} \sqrt[3]{1 - \frac{1}{x^2}}\, dx$$

$$y = 1 - \frac{1}{x^2} \to dy = \frac{2x}{x^4} dx \to dy$$

$$= \frac{2}{x^3} dx \to dx = \frac{x^3}{2} dy$$

$$= \int \frac{1}{\cancel{x^3}} \sqrt[3]{y}\, \frac{\cancel{x^3}}{2} dy$$

$$= \frac{1}{2} \times \frac{y^{\frac{4}{3}}}{\frac{4}{3}} + c = \frac{3}{8} \sqrt[3]{\left(1 - \frac{1}{x^2}\right)^4} + c$$

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28. $\int \frac{1 + \tan^2 x}{\tan x}\, dx$

$$= \int \frac{1}{\tan x} + \frac{\tan^2 x}{\tan x}\, dx$$

$$= \int \cot x\, dx + \int \tan x\, dx$$

$$= \ln|\sin x| - \ln|\cos x| + c$$

24. $\int \frac{1}{\sin x \cos x}\, dx$

$$= \int \frac{\sin^2 x + \cos^2 x}{\sin x \cos x}\, dx$$

$$= \int \frac{\sin^{\cancel{2}} x}{\cancel{\sin x} \cos x} + \frac{\cos^{\cancel{2}} x}{\sin x \cancel{\cos x}}\, dx$$

$$= \int \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\, dx$$

$$= -\ln|\cos x| + \ln|\sin x| + c$$

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25. $\int x^2 \times 3^{x^3}\, dx$

$$y = 3^{x^3} \to dx = \frac{dy}{3x^2 \ln 3 \times y}$$

$$\int \cancel{x^2 y} \times \frac{dy}{3\cancel{x^2} \ln 3 \times \cancel{y}}$$

$$= \int \frac{1}{3 \ln 3}\, dy$$

$$= \frac{1}{3 \ln 3} y + c$$

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26. $\int \sqrt{\frac{(x-5)^5}{x^9}}\, dx$

$$= \int \sqrt{\left(\frac{x-5}{x}\right)^5 \times \frac{1}{x^4}}\, dx$$

$$= \int \frac{1}{x^2} \sqrt{\left(1 - \frac{5}{x}\right)^5}\, dx$$

$$y = 1 - \frac{5}{x} \to dy = \frac{5}{x^2} dx \to dx = \frac{x^2}{5} dy$$

29. $\int \sin 3x \cos 6x \; dx$

$= \int \sin 3x \cos 2(3x) \; dx$

$= \int \sin 3x \, (2\cos^2(3x) - 1) \, dx$

$y = \cos 3x \rightarrow dy = -3 \sin 3x \, dx$

$\rightarrow dx = \dfrac{dy}{-3 \sin 3x}$

$= \int \cancel{\sin 3x}(2y^2 - 1) \dfrac{dy}{-3\cancel{\sin 3x}}$

$= -\dfrac{1}{3}\int (2y^2 - 1)dy$

$= -\dfrac{1}{3}\int \left(\dfrac{2y^3}{3} - y\right) + c$

$= -\dfrac{1}{3}\int \left(\dfrac{2}{3}\cos^3 3x - \cos 3x\right) + c$

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30. $\int \tan x \ln \cos x \, dx$

$y = ln\, cos\, x \rightarrow \dfrac{dy}{dx} = \dfrac{-\sin x}{\cos x} = -\tan x$

$\rightarrow dx = \dfrac{dy}{-\tan x}$

$\int \cancel{\tan x} \; y \dfrac{dy}{-\cancel{tan\, x}} = -\dfrac{y^2}{2} + c$

$= \dfrac{-(\ln \cos x)^2}{2} + c$

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31. $\int \dfrac{1 + \sin x}{\cos x \, (1 - \sin x)} dx$

$y = \sin x \rightarrow dx = \dfrac{dy}{\cos x}$

$\int \dfrac{1 + y}{cos\, x \, (1 - y)} \times \dfrac{dy}{\cos x}$

$= \int \dfrac{1 + y}{cos^2 x \, (1 - y)} \, dy$

$= \int \dfrac{1 + y}{1 - sin^2 x \, (1 - y)} \, dy$

$= \int \dfrac{\cancel{1 + y}}{(\cancel{1 - y^2})(1 - y)} \, dy$

$= \int \dfrac{1}{(1 - y)^2} \, dy = \int (1 - y)^{-2} \, dy$

$= \dfrac{(1 - y)^{-1}}{-1 \times -1} + c = \dfrac{1}{1 - \sin x} + c$

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32. $\int_1^2 \sqrt{1 - x^{\frac{-2}{3}}} \; dx$

$= \int_1^2 \sqrt{1 - \dfrac{1}{x^{\frac{2}{3}}}} \, dx = \int_1^2 \sqrt{\dfrac{x^{\frac{2}{3}} - 1}{x^{\frac{2}{3}}}} \, dx$

$= \int_1^2 \sqrt{\dfrac{\left(x^{\frac{1}{3}}\right)^2 - 1}{\left(x^{\frac{1}{3}}\right)^2}} \, dx$

36. $\int \sqrt{1+e^x}\,dx$

$y=\sqrt{1+e^x} \rightarrow y^2=1+e^x$

$\rightarrow 2y\,dy=e^x\,dx$

$\rightarrow \frac{2y\,dy}{e^x}=dx$

$\int y\,\frac{2y\,dy}{e^x} \rightarrow y^2-1$

$=\int \frac{2y^2}{y^2-1}\,dy$ → قسمة ثم كسور جزئية

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37. $\int \frac{1}{x\sqrt{x^{12}+x^{16}}}\,dx$

$=\int \frac{1}{x\sqrt{x^{16}\times\left(\frac{1}{x^4}+1\right)}}\,dx$

$=\int \frac{1}{x^5\sqrt{\frac{1}{x^4}+1}}\,dx$

افرض : $y=\frac{1}{x^4}+1$ → أكمل

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38. $\int_1^4 \frac{1}{\sqrt{x}(\sqrt{x}+4)}\,dx$

$y=\sqrt{x}$

$dy=\frac{1}{2\sqrt{x}}\,dx$

$dx=2y\,dy$

$x=1\rightarrow y=1$

$x=4\rightarrow y=4$

$\int_1^2 \frac{1}{y(y+4)}\,2y\,dy$

$\int_1^2 \frac{2}{(y+4)}\,dy$

$=2\ln y+4]_1^2=2\ln 6-2\ln 5$

$=\int_1^2 \frac{1}{x^{\frac{2}{3}}}\sqrt{x^{\frac{2}{3}}-1}\,dx$

افرض : $y=x^{\frac{2}{3}}-1$ → أكمل

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33. $\int \frac{1-2\sin^2 x}{1+\sin 2x}\,dx$

$=\frac{1}{2}\int \frac{2\cos 2x}{1+\sin 2x}\,dx$ → نشتق $2cos2x$

$=\frac{1}{2}\ln|1+\sin 2x|+c$

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34. $\int \frac{-1}{x^2}\ln\left(\frac{1}{x}\right)\,dx$

$y=\frac{1}{x}\rightarrow \frac{dy}{dx}=-\frac{1}{x^2}\rightarrow dx=-x^2dy$

$\int \frac{-1}{x^2}\ln y(-x^2)\,dy$

$=\int \ln y\,dy$ → أجزاء

$=y\,ln\,y-y+c=\frac{1}{x}\ln\left(\frac{1}{x}\right)-\frac{1}{x}+c$

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35. $\int x\sqrt{1-\cos 2x}\,dx \quad , \quad 0<x<\frac{\pi}{2}$

$=\int x\sqrt{2\sin^2 x}\,dx$

$=\sqrt{2}\int x\,\sin x\,dx$ → أجزاء

$$\int_0^{\frac{\pi}{2}} \sqrt{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^2}\ dx$$

$$\int_0^{\frac{\pi}{2}} \left|\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right|\ dx$$

$$\int_0^{\frac{\pi}{2}} \left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\ dx \rightarrow \text{أكمل}$$

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42. $\int \sin^5 x\ dx$

$$\int \sin x \sin^4 x\ dx$$

$$= \int \sin x\,(1-\cos^2 x)^2\ dx$$

$$y = \cos x \rightarrow dy = -\sin x\ dx$$
$$\rightarrow dx = \frac{dy}{-\sin x}$$

$$= \int \cancel{\sin x}\,(1-y^2)^2\ \frac{dy}{-\cancel{\sin x}}$$

$$= -\int 1-2y^2+y^4\ dy$$

$$= -\left(y-\frac{2y^3}{3}+\frac{y^5}{5}\right)+c$$

$$= -\left(\cos x-\frac{2\cos^3 x}{3}+\frac{\cos^5 x}{5}\right)+c$$

39. $\int \frac{1}{2+2\sin x}\ dx$

$$\int \frac{1}{2(1+\sin x)}\ dx \rightarrow \text{مرافقه} \times \frac{1-\sin x}{1-\sin x}$$

$$\int \frac{1-\sin x}{2(1-\sin^2 x)}\ dx = \frac{1}{2}\int \frac{1-\sin x}{\cos^2 x}\ dx$$

$$= \int \sec^2 x - \frac{\sin x}{\sin x \cos x}\ dx$$

$$= \tan x - \int \tan x \sec x\ dx$$
$$= \tan x - \sec x + c$$

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40. $\int \frac{x^3}{1-x^8}\ dx$

$$y = x^4$$
$$dy = 4x^3\ dx$$
$$dx = \frac{dy}{4x^3}$$

$$= \int \frac{x^3}{1-(x^4)^2}\ dx$$

$$= \int \frac{\cancel{x^3}}{1-y^2} \times \frac{dy}{\cancel{4x^3}}$$

$$= \frac{1}{4}\int \frac{1}{1-y^2}\ dy \rightarrow \text{كسور}$$

$$= \frac{1}{4}\left(\frac{1}{2}\ln|1-x^4|+\frac{1}{2}\ln|1+x^4|\right)+c$$

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41. $\int_0^{\frac{\pi}{2}} \sqrt{1-\sin x}\ dx$

$$= \int_0^{\frac{\pi}{2}} \sqrt{\sin^2\left(\frac{x}{2}\right)+\cos^2\left(\frac{x}{2}\right)+2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}\ dx$$

43. $\int \frac{x^2}{e^x}\,dx$

$= \int x^2 e^{-x}\,dx$

$z = x^2 \rightarrow dz = 2x\,dx$

$de = e^{-x}dx \rightarrow e = -e^{-x}$

$\int z\,.\,de = z.e - \int e\,.\,dz$

$= -x^2e^{-x} + \int 2xe^{-x}\,dx$

$= -x^2e^{-x} - 2xe^{-x} + 2\int e^{-x}\,dx$

$= -x^2e^{-x} - 2xe^{-x} - 2e^{-x} + c$

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44. $\int \frac{\left(1-\frac{1}{x}\right)^5}{x^2}\,dx$

$y = 1 - \frac{1}{x} \rightarrow dy = \frac{1}{x^2}\,dx \rightarrow dx = x^2\,dy$

$\int \frac{1}{\cancel{x^2}}\,y^5 \times \cancel{x^2}\,dy$

$= \frac{y^6}{6} + c = \frac{\left(1-\frac{1}{x}\right)^6}{6} + c$

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45. $\int \frac{1}{\sqrt{9+e^x}}\,dx$

$y = \sqrt{9+e^x}$

$y^2 = 9 + e^x$

$2y\,dy = e^x\,dx$

$\frac{2y\,dy}{e^x} = dx$

$\int \frac{1}{y} \times \frac{2y\,dy}{y^2-9}$

$\int \frac{1}{y^2-9}\,dy \rightarrow$ كسور

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46. $\int \left(\frac{x-1}{x+1}\right)^2 dx$

$= \int \left(1 + \frac{-2}{x+1}\right)^2 dx$

$x+1 \;\overline{)\; x-1}$ ، 1 ، $-x+1$ ، $2-$

$= \int 1 + \frac{4}{x+1} + \frac{4}{(x+1)^2}\,dx$

$= \int 1 + \frac{4}{x+1} + 4(x+1)^{-2}\,dx$

$= x - 4\ln|x+1| + \frac{4(x+1)^{-1}}{-1} + c$

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47. $\int \sec x\,dx$

$= \int sec\,x \times \frac{\sec x + tan\,x}{\sec x + \tan x}\,dx$

$= \int \frac{sec^2\,x + \sec x\,tan\,x}{sec\,x + tan\,x}\,dx$

$= \ln|\sec x + \tan x| + c$

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48. $\int \frac{dx}{x\,ln\,x \times \ln(\ln x)}$

$y = ln\,x \rightarrow \frac{dy}{dx} = \frac{1}{x}\,dx \rightarrow dx = x.dy$

$\int \frac{\cancel{x}.dy}{\cancel{x}\,y \times \ln y} = \int \frac{1}{y \times ln\,y}$

$w = \ln y \rightarrow \frac{dw}{dy} = \frac{1}{y} \rightarrow dy = y.dw$

$\int \frac{\cancel{y}\,dw}{\cancel{y}w} = \int \frac{1}{w}\,dw$

$= \ln|w| + c = \ln|\ln y| + c$

$= \ln\ln\ln x + c$

49. $\int \frac{\sin^6 x}{\cos^8 x}\,dx$

$\int \frac{\sin^6 x}{\cos^6 x} \times \frac{1}{\cos^2 x}\,dx$

$= \int \tan^6 x \times \sec^2 x\,dx$

$y = \tan x \rightarrow dx = \frac{dy}{sec^2 x}$

$\int y^6 \times \cancel{sec^2 x}\,\frac{dy}{\cancel{sec^2 x}}$

$= \frac{y^7}{7} + c = \frac{\tan^7 x}{7} + c$

50. $\int x^5 \left(2 - \frac{1}{x}\right)^4 dx$

$\int x\, x^4 \left(2 - \frac{1}{x}\right)^4 dx$

$= \int x \cancel{x^4} \frac{(2x-1)^4}{\cancel{x^4}}\,dx$

$= \int x(2x-1)^4\,dx \rightarrow$ أجزاء

51. $\int \frac{1 - 2\sin^2 x}{1 + \sin 2x}\,dx$

$y = 1 + \sin 2x \rightarrow dy = 2\cos 2x\,dx$

$\rightarrow dx = \frac{dy}{2\cos 2x}$

$\int \frac{\cancel{\cos 2x}}{y} \frac{dy}{2\cancel{\cos 2x}}$

$= \frac{1}{2}\ln|y| + c = \frac{1}{2}\ln|1 + \sin 2x| + c$

52. $\int \frac{1}{x - \sqrt{x}}\,dx$

$y = \sqrt{x} \rightarrow y^2 = x \rightarrow dy = \frac{1}{2\sqrt{x}}dx$

$\rightarrow dy = \frac{1}{2y}dx \rightarrow dx = 2y\,dy$

$\int \frac{1}{y^2 - y} 2y\,dy = \int \frac{2\cancel{y}}{\cancel{y}(y-1)}\,dy$

$= 2\ln|y-1| + c = 2\ln|\sqrt{x} - 1| + c$

53. $\int \sin\sqrt{x}\,dx$

$y = \sqrt{x} \rightarrow y^2 = x \rightarrow 2y\,dy = 1dx$

$\int \sin y\ 2ydy = 2\underbrace{\int y \sin y\,dy}$

أجزاء

$z = y \rightarrow dz = 1\,dy$

$de = \sin y\,dy \rightarrow e = -\cos y$

$\int z.de = z.e - \int e.dz$

$= -y\cos y + \int \cos y\,dy$

$= -y\cos y + \sin y + c$

$= -\sqrt{x}\cos\sqrt{x} + \sin\sqrt{x} + c$

54. $\int \frac{e^{2x}}{\sec x}\,dx$

$= \int e^{2x}\cos x\,dx$

$z = e^{2x} \rightarrow dz = 2e^{2x}\,dx$

$de = \cos x\,dx \rightarrow e = \sin x$

$$= \int \cancel{sec^2 x}\,(y^2+1)^2 \frac{dy}{\cancel{sec^2 x}}$$

$$= \int (y^4+2y^2+1)\,dy \;\rightarrow\; \text{أكمل}$$

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57. $\int (x^6-4x)^4 dx$

$$= \int \left(x(x^5-4)\right)^4 dx$$

$$= \int x^4(x^5-4)^4 dx$$

$$y = x^5-4 \rightarrow dx = \frac{dy}{5x^4}$$

$$= \int \cancel{x^4} y^4 \frac{dy}{5\cancel{x^4}}$$

$$= \frac{1}{5}\times\frac{y^5}{5}+c = \frac{1}{25}(x^5-4)^5+c$$

---

58. $\int \left(\frac{\tan x-1}{1+\tan x}\right)^5 sec^2\left(x-\frac{\pi}{4}\right) dx$

$$= \int \left(\frac{tan\,x-\tan\frac{\pi}{4}}{1+tan\,x\,tan\frac{\pi}{4}}\right)^5 sec^2\left(x-\frac{\pi}{4}\right) dx$$

$$= \int \tan\left(x-\frac{\pi}{4}\right)^5 sec^2\left(x-\frac{\pi}{4}\right) dx$$

$$y = \tan\left(x-\frac{\pi}{4}\right) \rightarrow dx = \frac{dy}{\tan^2\left(x-\frac{\pi}{4}\right)}$$

$$= \int y^5 \frac{\cancel{sec^2\left(x-\frac{\pi}{4}\right)}\,dy}{\cancel{sec^2\left(x-\frac{\pi}{4}\right)}}$$

$$= \frac{y^6}{6}+c = \frac{tan^6\left(x-\frac{\pi}{4}\right)}{6}+c$$

$$\int z.de = z.e - \int e.dz$$

$$\int e^{2x}\cos x\,dx = e^{2x}\sin x - 2\int e^{2x}\sin x\,dx$$

$$\int e^{2x}\cos x\,dx$$ أجزاء

$$= e^{2x}\sin x - 2\left(e^{2x}\cos x + 2\int e^{2x}\cos x\,dx\right)$$

$$= e^{2x}\sin x + 2e^{2x}\cos x - 4\int e^{2x}\cos x\,dx$$

$$\frac{5}{5}\int e^{2x}\cos x\,dx = \frac{e^{2x}\sin x + 2e^{2x}\cos x}{5}+c$$

---

55. $\int \frac{1}{(\cos x+\sin x)^2} dx$

$$= \int \frac{1}{\left(\cos x\left(1+\frac{\sin x}{\cos x}\right)\right)^2} dx$$

$$= \int \frac{1}{\cos^2 x\,(1+\tan x)^2} dx$$

$$= \int \frac{\sec^2 x}{(1+\tan x)^2} dx$$

افرض : $y = \tan x \;\rightarrow\;$ أكمل

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56. $\int sec^6 x\,dx$

$$\int sec^2 x\,sec^4 x\,dx$$

$$= \int sec^2 x\,(\tan^2 x+1)^2 dx$$

$$y = \tan x \rightarrow dx = \frac{dy}{\sec^2 x}$$

59. $\int x(\ln x)^2\,dx$

$z=(\ln x)^2 \to dz=2\ln x\times\frac{1}{x}dx$

$de=x\to e=\frac{x^2}{2}$

$=\frac{x^2}{2}(\ln x)^2-\int x\ln x\,dx$

$=\frac{x^2}{2}(\ln x)^2-\left(\frac{x^2}{2}\ln x-\int\frac{x}{2}\,dx\right)$

$=\frac{x^2}{2}(\ln x)^2-\frac{x^2}{2}\ln x+\frac{x^2}{4}+c$

60. $\int\frac{1}{x+3x\sqrt[4]{x}}dx$

$y=\sqrt[4]{x}\to y^4=x\to 4y^3dy=1\,dx$

$\int\frac{1}{y^4+3y^4y}4y^3\,dy$

$=\int\frac{4y^3}{y^4+3y^5}dy=\int\frac{4\cancel{y^3}}{\cancel{y^3}(y+3y^2)}dy$

$=\int\frac{4}{y+3y^2}dy$ → كسور جزئية

61. $\int\frac{1}{x^8-x}dx$

$y=1-\frac{1}{x^7}$

$dy=\frac{7}{x^8}dx$

$dx=\frac{x^8}{7}dy$

اخرج (x^8) عامل مشترك

$\int\frac{1}{x^8\left(1-\frac{1}{x^7}\right)}dx$

$=\int\frac{1}{\cancel{x^8}y}\times\frac{\cancel{x^8}}{7}\,dy=\frac{1}{7}\int\frac{1}{y}dy$

$=\frac{1}{7}\ln|y|c=\frac{1}{7}\ln\left|1-\frac{1}{x^7}\right|+c$

62. $\int x\cos\sqrt{1+x^2}\,dx$

$y=\sqrt{1+x^2}$

$y^2=1+x^2$

$2y\,dy=2x\,dx$

$\frac{2ydy}{2x}=dx$

$\int\cancel{x}\cos y\frac{ydy}{\cancel{x}}$

$\int y\cos y\,dy$

63. $\int\frac{\sin^4 x}{\cos^2 x}dx$

$\int\frac{(\sin^2 x)^2}{\cos^2 x}dx=\int\frac{(1-\cos^2 x)^2}{\cos^2 x}dx$

$=\int\frac{1-2\cos^2 x+\cos^4 x}{\cos^2 x}dx$

$=\int(\sec^2 x-2+\cos^2 x)\,dx$

$=\int\left(\sec^2 x-2+\frac{1}{2}(1+\cos 2x)\right)dx$

$=\tan x-2x+\frac{1}{2}\left(x+\frac{\sin 2x}{2}\right)+c$

64. $\int\frac{\cos x}{\sin x\ln\sin x}dx$

$\frac{\cos x}{\sin x}=\cot x$

$=\int\frac{\cancel{\cot x}}{y}\times\frac{dy}{\cancel{\cot x}}$

$=\int\frac{1}{y}.dy$

$=\ln|y|+c$

$=\ln|\ln\sin x|+c$

$y=\ln\sin x$

$\frac{dy}{dx}=\frac{\cos x}{\sin x}=\tan x$

$dx=\frac{dy}{\cot x}$

65. $\int \frac{\sin x}{\csc^2 x}\, dx$

$= \int \sin x \sin 2x \; dx$

$= \int \sin x \times 2 \sin x \cos x \; dx$

$= \int 2 \sin^2 x \cos x \; dx$

افرض $y = \sin x \rightarrow$ أكمل

66. $\int \frac{\sin^2 3x + 2 \sin 3x + 1}{\sec 3x}\, dx$

$= \int (\sin^2 3x + 2\sin 3x + 1) \cos 3x \; dx$

$= \int \sin^2 3x \cos 3x \; dx + 2 \int \sin 3x \cos 3x \; dx + \int \cos 3x \; dx$

$y = \sin 3x \rightarrow dy = 3 \cos 3x \, dx$
$\rightarrow dx = \frac{dy}{3 \cos 3x}$

$= \int y^2 \cancel{\cos 3x} \frac{dy}{3 \cancel{\cos 3x}} + \cancel{2} \int \frac{\sin 6x}{\cancel{2}}\, dx + \frac{\sin 3x}{3}$

$= \frac{y^3}{3} - \frac{\cos 6x}{6} + \frac{\sin 3x}{3} + c$

$= \frac{\sin^3 3x}{9} - \frac{\cos 6x}{6} + \frac{\sin 3x}{3} + c$

67. $\int x^3 \times \csc^2 x^2 \; dx$

$y = x^2 \rightarrow dy = 2x \, dy \rightarrow dx = \frac{dy}{2x}$

$\int x^{\cancel{3}\,2} \times \csc^2 y \frac{dy}{2\cancel{x}} = \frac{1}{2} \int y \times csc^2 y \; dy$

$z = y \rightarrow dz = 1 \, dy$

$de = csc^2 y \, dy \rightarrow e = -\cot y$

$\frac{1}{2} \int y \, csc^2 y \; dy = \frac{1}{2} \left(-y \csc y + \int coty \, dy \right)$

$= \frac{1}{2} (-y \, coty + \ln|sin \, y|) + c$

$= \frac{-1}{2} x^2 \csc x^2 + \frac{1}{2} \ln|\sin x^2| + c$

68. $\int \frac{\sqrt{x^{-1} + 4}}{x^2}\, dx$

$\int \frac{1}{x^2} \sqrt{\frac{1}{x} + 4}\, dx$

$y = \frac{1}{x} + 4 \rightarrow dy = -\frac{1}{x^2} dx$
$\rightarrow dx = -x^2 \, dy$

$\int \frac{-1}{\cancel{x^2}} \sqrt{y} \cancel{x^2}\, dy$

$= -\frac{y^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{-2}{3} \sqrt{\left(\frac{1}{x} + 4\right)^3} + c$

$$= \int \frac{\cancel{\sin^2 x}\ \sin x}{\cancel{\sin x}\cos x} + \frac{\cancel{\cos^2 x}\ \cancel{\cos x}}{\sin x \cancel{\cos x}}\ dx$$

$$= \int \frac{\sin x}{\cos x} dx + \int \frac{\cos x}{\sin x}\ dx$$

$$= -\ln|\cos x| + \ln|\sin x| + c$$

---

73. $\int \sec^3 x\ dx$

$$= \int \sec x \sec^2 x\ dx$$

$$= \int \sec x\,(\tan^2 x + 1)\ dx$$

$$= \int \underbrace{\tan x}_{z} \times \underbrace{\tan x \sec x\ dx}_{de} + \int \sec x\ dx$$

$z = \tan x \to dz = \sec^2 x$

$de = \tan x \sec x \to e = \sec x$

$$\int \sec^3 x = \tan x \sec x - \int \sec^3 x\ dx + \int \sec x\ dx$$

$$2\int \sec^3 x = \tan x \sec x + \int \sec x\ dx$$

$$\int \sec^3 x = \frac{\tan x \sec x}{2} + \frac{1}{2}\ln|\sec x + \tan x| + c$$

69. $\int \frac{\cos 2x}{\sin x \cos x}\ dx$

$$= \int \frac{\cos 2x}{\left(\frac{\sin 2x}{2}\right)}\ dx = \int \frac{2\cos 2x}{\sin 2x}\ dx$$

$$= \ln|\sin 2x| + c$$

---

70. $\int \frac{1}{\sqrt[4]{x} - x}\ dx$

$y = \sqrt[4]{x} \to y^4 = x \to 4y^3 dy = dx$

$$\int \frac{1}{y - y^4} 4y^3\ dy$$

$$= \int \frac{4\cancel{y^3}^{\,2}}{\cancel{y}(1 - y^3)}\ dy$$

$$= \int \frac{4y^2}{(1 - y^3)}\ dy = \frac{4}{-3}\int \frac{-3y^2}{1 - y^3}\ dy$$

$$= -\frac{4}{3}\ln|1 - y^3| + c = \frac{-4}{3} ln\left|1 - \left(\sqrt[4]{x}\right)^3\right| + c$$

---

71. $\int \frac{1}{\sqrt{x+1} - \sqrt{x}}\ dx$

$$= \int \frac{1}{\sqrt{x+1} - \sqrt{x}} \times \frac{\sqrt{x+1} + \sqrt{x}}{\sqrt{x+1} + \sqrt{x}} dx$$

$$= \int \frac{\sqrt{x+1} + \sqrt{x}}{\cancel{x} + 1 - \cancel{x}}\ dx$$

$$= \int (1+x)^{\frac{1}{2}} + x^{\frac{1}{2}}\ dx \to \text{أكمل}$$

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72. $\int \frac{1}{\sin x \cos x}\ dx$

$$= \int \frac{\sin^2 x + \cos^2 x}{\sin x \cos x}\ dx$$

74. $\int_1^2 \sqrt{1-x^{-\frac{2}{3}}}\,dx$

$= \int_1^2 \sqrt{1-\frac{1}{x^{\frac{2}{3}}}}\,dx = \int_1^2 \sqrt{\frac{x^{\frac{2}{3}}-1}{x^{\frac{2}{3}}}}\,dx$

$= \int_1^2 \frac{1}{x^{\frac{1}{3}}}\sqrt{x^{\frac{2}{3}}-1}\,dx$

$y = x^{\frac{2}{3}} - 1$

$dy = \frac{2}{3}x^{-\frac{1}{3}}\,dx$

$dx = \frac{3}{2}x^{\frac{1}{3}}\,dy$

$= \frac{3}{2}\int_1^2 \frac{1}{x^{\frac{1}{3}}}\sqrt{y}\,x^{\frac{1}{3}}\,dy$

$= \frac{3}{2}\times\frac{y^{\frac{3}{2}}}{\frac{3}{2}} = \sqrt{\left(x^{\frac{2}{3}}-1\right)^3}\Big|_1^2$

$= \sqrt{\left(\sqrt[3]{4}-1\right)^3} - \sqrt{\left(\sqrt[3]{1}-1\right)^3}$

$= \sqrt{\left(\sqrt[3]{4}-1\right)^3}$

---

75. $\int \frac{1}{\sqrt{x}\sin^2\sqrt{x}}\,dx$

$= \int \frac{1}{\sqrt{x}}\csc^2\sqrt{x}\,dx$

$y = \sqrt{x} \to y^2 = x \to 2y\,dy = dx$

$= \int \frac{1}{y}\times\csc^2 2y\,dy$

$= -2\cot y + c = -2\cot\sqrt{x} + c$

---

76. $\int \frac{\sqrt{1+\cos 2x}}{-\cos^3 x}\times\tan^3 x\,dx$

$= \int \frac{\sqrt{2\cos^2 x}}{-\cos^3 x}\times\tan^3 x\,dx$

$= \int \frac{-\sqrt{2}\,\cancel{\cos x}}{\cos^{\cancel{3}\,2} x}\times\tan^3 x\,dx$

$= \int -\sqrt{2}\sec^2 x\times\tan^3 x\,dx$

$y = \tan x \to dx = \frac{dy}{\sec^2 x}$

$= \int -\sqrt{2}\,\cancel{\sec^2 x}\times y^3\frac{dy}{\cancel{\sec^2 x}}$

$= -\frac{\sqrt{2}}{4}y^4 + c = -\frac{\sqrt{2}}{4}(\tan^4 x) + c$

---

77. $\int \frac{1+\sqrt{e^x}}{e^x}\,dx$

$= \int \frac{1}{e^x} + \frac{e^{\frac{1}{2}x}}{e^x}\,dx = \int e^{-x} + e^{\frac{-1}{2}x}\,dx$

$= \frac{e^{-x}}{-1} + \frac{e^{\frac{-1}{2}x}}{\frac{-1}{2}} + c$

---

78. $\int \frac{x^2}{x^2-(2+x)^2}\,dx$

$-\frac{1}{4}+\frac{1}{4}$

$-4x-4 \;\big|\; x^2$

$-x^2 \mp x$

$-x$

$+ -x - 1$

1

$= \int \frac{x^2}{x^2-(4+4x+x^2)}\,dx$

$= \int \frac{x^2}{\cancel{x^2}-4-4x-\cancel{x^2}}\,dx$

$$= \int \frac{6y^5}{y^2 - y^3}\, dy = \int \frac{6y^{\cancel{5}\ 3}}{\cancel{y^2}(1-y)}\, dy$$

$$= \int \frac{6y^3}{1-y}\, dy \rightarrow \text{قسمة طويلة}$$

---

82. $\int \sin x\,(x + \csc^3 x)\, dx$

$$= \int x \sin x + \cancel{\sin x} \times \frac{1}{\sin^{\cancel{3}\ 2} x}\, dx$$

$$= \int \underbrace{x \sin x}_{\text{أجزاء}} + \underbrace{\csc^2 x\ dx}_{\text{مباشر}}$$

---

83. $\int e^{x+e^x}\, dx$

$$= \int e^x e^{e^x}\, dx$$

$$= \int \cancel{y}\, e^y\, \frac{dy}{\cancel{y}}$$

$$= e^y + c = e^{e^x} + c$$

$y = e^x$
$dy = e^x dx$
$dx = \frac{dy}{e^x}$

تمارين وواجبات

1. $\int \frac{x \sin x}{\cos^3 x}\, dx$

2. $\int \frac{x^2 + x + 5}{x^2 + x}\, dx$

3. $\int \sqrt[3]{2x^5 - x^3}\, dx$

4. $\int \frac{7\, dx}{x^2 - 3x - 10}$

$$= \int \frac{x^2}{-4 - 4x}\, dx$$

$$= \int \left(-\frac{1}{4}x + \frac{1}{4}\right) dx + \int \frac{1}{-4x - 4}\, dx$$

$$= -\frac{1}{4} \times \frac{x^2}{2} + \frac{1}{4}x + \frac{-1}{4} \ln|-4x - 4| + c$$

---

79. $\int \frac{1}{e^x - e^{-x}}\, dx$

$$= \int \frac{1}{e^x - \frac{1}{e^x}}\, dx$$

$$= \int \frac{1}{\frac{e^{2x} - 1}{e^x}}\, dx = \int \frac{e^x}{e^{2x} - 1}\, dx$$

$$y = e^x \rightarrow dy = e^x\, dx \rightarrow dx = \frac{dy}{e^x}$$

$$= \int \frac{\cancel{y}}{y^2 - 1} \times \frac{dy}{\cancel{y}} = \int \frac{1}{y^2 - 1} dy \rightarrow \text{كسور جزئية}$$

$$= \frac{1}{2} \ln|e^x - 1| - \frac{1}{2} \ln|e^x + 1| + c$$

---

80. $\int \sqrt{1 + \sqrt{x}}\, dx$

$$y = \sqrt{x} \rightarrow y^2 = x \rightarrow 2y\, dy = dx$$

$$\int \sqrt{1 + y}\ 2y dy = \int 2y(1 + y)^{\frac{1}{2}}\, dy$$

---

81. $\int \frac{1}{\sqrt[3]{x} - \sqrt{x}}\, dx$

$$x = y^6 \rightarrow dx = 6y^5\, dy$$

$$\int \frac{1}{\sqrt[3]{y^6} - \sqrt{y^6}} 6y^5\, dy$$

5. $\int \frac{(x+1)^5}{x^7}\,dx \;, x \neq 0$

6. $\int \frac{2x^2}{1-x^2}\,dx \;, x \neq \pm 1$

7. $\int_1^3 \frac{7}{x^2+4x+4}\,dx$

8. $\int x\,sec^2 x\;dx$

9. $\int \frac{\sqrt{x+1}+1}{\sqrt{x+1}-1}\,dx$

10. $\int (\cos x + \sin x)^2\,dx$

11. $\int_0^2 \sqrt{x^2-2x+1}\,dx$

12. $\int \frac{\sqrt{x}}{x-9}\,dx$

13. $\int (x^5-3x)^3\,dx$

14. $\int x^5 \cos x^3\;dx$

15. $\int \frac{1}{e^x-1}\,dx$

16. $\int \frac{3}{x^2+x-12}\,dx$

17. $\int e^x \cos x\;dx$

18. $\int \frac{1}{x^2}\sqrt{\frac{2x+1}{x}}\,dx$

19. $\int \frac{\cos 2x}{\sin x \cos x}\,dx$

20. $\int_1^e (\ln x)^2\,dx$

21. $\int \frac{(1+x)^9}{x^{11}}\,dx$

22. $\int x^n \ln x\;dx$

23. $\int \sin 2x\, e^{\cos x}\,dx$

24. $\int \sin^3 x \cos^3 x\;dx$

25. $\int \frac{3x-1}{x^2+x-2}\,dx$

26. $\int x^5\sqrt{x^3+1}\,dx$

27. $\int \frac{2x-3}{x^2-3x-4}\,dx$

28. $\int x(\ln x)^2\,dx$

29. $\int \sin x \cos^2(\cos x)\;dx$

30. $\int x^3 e^{x^2+1}\,dx$

31. $\int \frac{1}{2x-1+\sqrt{2x+1}}\,dx$

32. $\int_0^1 x(1+x^2)^4\,dx$

33. $\int \frac{\ln(x+2)}{\sqrt{x+2}}\,dx$

34. $\int \frac{7x}{2x^2-5x-3}\,dx$

35. $\int x\tan^2 x\;dx$

36. $\int \frac{1}{x+2x\sqrt[3]{x}}\,dx$

37. $\int_0^4 x\sqrt{x^2+9}\,dx$

38. $\int \frac{6\cos x}{(2\sin x-1)(\sin x+1)}\,dx$

39. $\int \frac{xe^x}{(1+x)^2}\,dx$

40. $\int x(\sin x+\cos x)^2\,dx$

41. $\int e^{x+3e^x}\,dx$

42. $\int \frac{2x^2-x^4}{x^2}\,dx$

43. $\int (x^2+1)\cos(x^3+3x+1)\;dx$

44. $\int x^2\ln x\;dx$

45. $\int x\cos\sqrt{1+x^2}\;dx$

46. $\int \frac{\sqrt{x+2}}{x+1}\,dx$

47. $\int \frac{\sqrt{x}}{\sqrt{x}+2}\,dx$

48. $\int \frac{\sqrt{x+1}}{x}\,dx$

49. $\int \sqrt{1-e^{-x}}\,dx$

50. $\int \frac{\sqrt{4x+20}-3}{\sqrt{x+5}+3}\,dx$

51. $\int_0^2 \frac{1}{x^2-4}\,dx$

52. $\int \frac{2x+1}{x^2+3x-4}\,dx$

المعادلات التفاضلية

النوع الأول : حل المعادلة التفاضلية

أفصل المتغير (x) مع تفاضلته (dx) عن المتغير (y) مع تفاضلته (dy) ثم نكامل الطرفين

أمثلة

حل المعادلات التفاضلية الآتية :

① $\frac{dy}{dx} = \frac{2x+5}{3y^2}$

نضرب ضرب تبادلي :

$3y^2\,dy = 2x+5\,dy$

نكامل الطرفين :

$\int 3y^2\,dy = \int 2x+5\,dy$

$\frac{3y^3}{3} = \frac{2x^2}{2} + 5x + c$

$y^3 = x^2 + 5x + c$

---

② $\frac{dy}{dx} = \sqrt{xy}\ , x > 0\ ,\ y > 0$

$\frac{dy}{dx} = \frac{\sqrt{xy}}{1}$

$\int \frac{dy}{\sqrt{y}} = \int \sqrt{x}\,dx$

$\int y^{-\frac{1}{2}}\,dy = \int x^{\frac{1}{2}}\,dx$

$\frac{2y^{\frac{1}{2}}}{1} = \frac{2x^{\frac{3}{2}}}{3} + c$

---

③ $x^2\,dx - y^2\,dy = \sin y\ dy + 8x\,dx$

$-y^2\,dy - \sin y\ dy = 8x\,dx - x^2\,dx$

$\int (-y^2 - \sin y)\,dy = \int (8x - x^2)\,dx$

$\frac{-y^3}{3} + \cos y = 4x^2 - \frac{x^3}{3} + c$

---

④ $\frac{dy}{dx} = \cos^2 y \sin 3x$

أقسم على $(\cos^2 y)$ مع ضرب تبادلي

$\frac{dy}{\cos^2 y} = \sin 3x\,dx$

$\int \sec^2 y\,dy = \int \sin 3x\,dx$

$\tan y = \frac{-1}{3}\cos 3x + c$

---

⑤ $3\,dy - y^3\,dx = 3\cos^2 x\ dy$

$3\,dy - 3\cos^2 x\ dy = y^3\,dx$

$3\,(1 - \cos^2 x)\,dy = y^3\,dx$

$\frac{dy}{y^3} = \frac{dx}{3\sin^2 x}$

$\int y^{-3}\,dy = \int \frac{1}{3}\csc^2 x\ dx$

$\frac{y^{-2}}{-2} = -\frac{1}{3}\cot x + c$

---

⑥ $\frac{2dy}{1-x} = (2x+1)dx$

$2\,dy = (2x+1)(1-x)\,dx$

$2\,dy = (2x - 2x^2 + 1 - x)\,dx$

⑧ $\frac{y}{dx} = \frac{\sec^4 x \tan^2 x}{dy}$

$\int y\,dy = \int sec^4 x\, tan^2 x\; dx$

$A = \tan x$

$dx = \frac{dw}{\sec^2 x}$

$\frac{y^2}{2} = \int sec^4 x\, tan^2 x\; dx$

$\frac{y^2}{2} = \int sec^4 x \times w^2 \times \frac{dw}{sec^2 x}$

$\frac{y^2}{2} = \int sec^2 x\, w^2\, dw$

$\frac{y^2}{2} = \int (1 + w^2)\, w^2\, dw$

$\frac{y^2}{2} = \int w^2 + w^4\, dw$

$\frac{y^2}{2} = \frac{w^3}{3} + \frac{w^5}{5} + c = \frac{\tan^3 x}{3} + \frac{\tan^5 x}{5} + c$

---

⑨ $\frac{dy}{dx} = \sqrt{xy + x + y + 1}$

$\frac{dy}{dx} = \sqrt{x(y+1) + (y+1)}$

$\frac{dy}{dx} = \sqrt{(y+1)(x+1)}$

$\frac{dy}{dx} = \sqrt{y+1}\,\sqrt{x+1}$

$\int \frac{dy}{\sqrt{y+1}} = \int \sqrt{x+1}\, dx$

$\int \frac{dy}{\sqrt{y+1}} = \int \sqrt{x+1}\, dx$

$\int (y+1)^{-\frac{1}{2}}\, dy = \int (x+1)^{\frac{1}{2}}\, dx$

$2(y+1)^{\frac{1}{2}} = \frac{2}{3}(x+1)^{\frac{3}{2}} + c$

$2\,dy = (-2x^2 + x + 1)\, dx$

$\int dy = \int \left(-x^2 + \frac{1}{2}x + \frac{1}{2}\right) dx$

$y = -\frac{1}{3}x^3 + \frac{1}{4}x^2 + \frac{1}{2}x + c$

---

⑦ $\sec^2 \frac{x}{4}\, dy - \sin^2 \frac{x}{4}\, dx = 0$

$\sec^2 \frac{x}{4}\, dy = sin^2 \frac{x}{4}\, dx$

$dy = \frac{sin^2 \frac{x}{4}}{sec^2 \frac{x}{4}}\, dx$

$dy = \sin^2 \frac{x}{4} \cos^2 \frac{x}{4}\, dx$

ونعلم أن : $\sin 2x = 2 \sin x \cos x$

$\rightarrow \frac{1}{2}\sin \frac{x}{2} = \sin \frac{x}{4} \cos \frac{x}{4}$

وربع الطرفين : $\frac{1}{2}\sin^2 \frac{x}{2} = \sin^2 \frac{x}{4} \cos^2 \frac{x}{4}$

$= \frac{1}{4} \times \frac{1}{2}(1 - \cos x)$

$\frac{1}{4}\sin^2 \frac{x}{2} = \frac{1}{8}(1 - \cos x)$

$\int dy = \int \frac{1}{8}(1 - cos\, x)\, dx$

$y = \frac{1}{8}(x - \sin x) + c$

⑩ $\frac{dy}{dx} = 2y$

$dy = 2y\,dx \rightarrow \int \frac{dy}{y} = \int 2\,dx$

$\ln y = 2x + c \rightarrow y = e^{2x} + c$

---

⑪ $\frac{dy}{dx} \times \frac{1}{x^2-4} = \frac{e^{-2y}(x+3)}{x^2-2x}$

$\frac{dy}{dx} = \frac{e^{-2y}(x+3)(x^2-4)}{x^2-2x}$

$\frac{dy}{e^{-2y}} = \frac{(x+3)\cancel{(x-2)}(x+2)dx}{x\cancel{(x-2)}}$

$e^{2y}dy = \frac{x^2+5x+6}{x}dx$

$\int e^{2y}dy = \int x+5+\frac{6}{x}\,dx$

$\frac{e^{2y}}{2} = \frac{x^2}{2} + 5x + 6\ln|x| + c$

---

⑫ $dy = \frac{y^2\,dx}{x^2-x}$

$\int \frac{dy}{y^2} = \int \frac{dx}{x^2-x}$

$\int y^{-2}\,dy = \int \frac{1}{x(x-1)}dx$

$\frac{y^{-1}}{-1} = \int \frac{a}{x} + \frac{b}{x-1}dx$ كسور جزئية

$1 = a(x-1) + b(x)$

$x = 1 \rightarrow b = 1 \rightarrow x = 0 \rightarrow a = -1$

$\frac{y^{-1}}{-1} = \int \frac{-1}{x} + \frac{1}{x-1}dx$

$\frac{y^{-1}}{-1} = -\ln|x| + \ln|x-1| + c$

⑬ إذا كانت $\frac{dy}{dx} = y^2e^{-x}$ وكانت $y = 2$

عندما $x = 0$, أثبت أن : $y = \frac{2e^x}{2-e^x}$

$\frac{dy}{dx} = y^2e^{-x} \rightarrow \int \frac{dy}{y^2} = \int e^{-x}\,dx$

$\frac{y^{-1}}{-1} = -e^{-x} + c \rightarrow \frac{-1}{y} = \frac{-1}{e^x} + c$

عندما $y = 2$ $\quad x = 0 \rightarrow c = \frac{1}{2}$

$\frac{-1}{y} = \frac{-1}{e^x} + \frac{1}{2} \quad \times -1$

$\frac{1}{y} = \frac{1}{e^x} - \frac{1}{2} \rightarrow y = \frac{2e^x}{2-e^x}$

النوع الثاني : ميل المماس

أمثلة

① إذا كان ميل المماس لمنحنى (dx) هو $5x^4 + 6x$ وكان $d(0) = 3$, احسب $d(x)$

نكامل الطرفين $d'(x) = 5x^4 + 6x$

$d(x) = \int 5x^4 + 6x\,dx$

$d(x) = \frac{5x^5}{5} + \frac{6x^2}{2} + c$

$d(x) = x^5 + 3x^2 + c$

التخلص من (c) $\quad d(0) = 3$

$3 = (0)^5 + 3(0)^2 + c \quad \rightarrow \quad c = 3$

$d(x) = x^5 + 3x^2 + 3$

② إذا كان ميل المماس لمنحنى d عند (x, y) هو $\frac{4x^3 + 2x}{2y}$ وكان d ويمر بالنقطة $(1, 3)$

, فما قاعدة الاقتران d

$$\frac{dy}{dx} = \frac{4x^3 + 2x}{2y}$$

$$\int 2y\, dy = \int 4x^3 + 2x\, dx$$

$$y^2 = x^4 + x^2 + c$$

التخلص من (c) $(1, 3)$

$$9 = 1 + 1 + c \rightarrow c = 7$$

$$y^2 = x^4 + x^2 + 7$$

---

③ إذا كان $w'' = 12x - 10$ وكانت $(1, 7)$

حرجة فما قاعدة الاقتران w

$$w'(x) = \int w''(x)\, dx$$

$$w'(x) = \int 12x - 10\, dx$$

$$w'(x) = 6x^2 - 10x + c$$

لكن $(1, 7)$ حرجة $\leftarrow w'(1) = 0 \leftarrow c = 4$

$$w'(x) = 6x^2 - 10x + 4$$

$$w(x) = \int w'(x)\, dx$$

$$w(x) = \int 6x^2 - 10x + 4\, dx$$

$$w(x) = 2x^3 - 5x^2 + 4x + c$$

$(1, 7)$ تحقق $\leftarrow 6 = c$

$$w(x) = 2x^3 - 5x^2 + 4x + 6$$

---

④ إذا كان ميل المماس لمنحنى العلاقة

$\frac{4x}{3y^2\sqrt{x^2 + 5}}$ وكان منحنى w يمر بالنقطة

$(2, 3)$, فما فقاعدة الاقتران

$$\frac{dy}{dx} = \frac{4x}{3y^2\sqrt{x^2 + 5}}$$

$$\int 3y^2\, dy = \int \frac{4x}{\sqrt{x^2 + 5}}\, dx$$

$$y^3 = \int (4x)(x^2 + 5)^{-\frac{1}{2}}\, dx$$

$$l = x^2 + 5$$

$$dx = \frac{dl}{2x}$$

$$y^3 = \int (4x)(l)^{-\frac{1}{2}} \frac{dl}{2x}$$

$$y^3 = \int 2l^{-\frac{1}{2}}\, dl$$

$$y^3 = 2 \times 2l^{\frac{1}{2}} + c = 4\sqrt{l} + c$$

$$y^3 = 4\sqrt{x^2 + 5} + c$$

$$(2, 3) \rightarrow 27 = 4\sqrt{9} + c$$

$$27 = 12 + c \rightarrow c = 15$$

---

⑤ إذا كان ميل المماس حسب العلاقة

$\frac{7}{2x - 3}$ وكان w يمر بالنقطة $(2, 5)$ فما قاعدة

الاقتران

$$w'(x) = \frac{7}{2x - 3}$$

$$w(x) = \int \frac{7}{2x-3}\,dx$$

$$= \frac{7}{2}\ln|2x-3| + c \text{ لكن } w(2) = 5$$

$$5 = \frac{7}{2}\ln 1 + c \rightarrow c = 5$$

$$\rightarrow w = \frac{7}{2}\ln|2x-3| + 5$$

النوع الثالث : الـفـيـزيـاء

أمـثـلـة

① إذا كان تسارع جسيم $t = 6n + 6$ احسب المسافة بعد مرور ثانيتين علما بأن السرعة الابتدائية $3m/sec$ والجسيم قطع $15m$ خلال ثانية واحدة من الحركة

$$w = \int t\,dn$$

$$w = \int 6n + 6\,dn = \frac{6n^2}{2} + 6n + c$$

$$w(0) = 3 \rightarrow 3 = 0 + 0 + c \rightarrow c = 3 \text{ لكن}$$

$$w = 3n^2 + 6n + 3 \text{ لكن } Q = \int w\,dn$$

$$= n^3 + 3n^2 + 3n + c$$

$$Q(1) = 15 \rightarrow 15 = 1 + 3 + 3 + c$$
$$\rightarrow c = 8$$

$$Q = n^3 + 3n^2 + 3n + 8$$

$$Q(2) = 8 + 12 + 6 + 8 = 34\,m$$

② قذف جسيم بتسارع $t = -10m/sec^2$ وكانت $w(0) = 50m/sec$ وكان الارتفاع عن سطح الأرض بعد ثانية واحدة يساوي (70) , فما معادلة الحركة

$$t = -10 \rightarrow w = \int -10dn \rightarrow w = -10n + c$$

$$w(0) = 50 \rightarrow c = 50 \rightarrow w = -10n + 50 \text{ لكن}$$

$$Q = \int w\,dn \rightarrow Q = \int -10n + 50\,dn$$

$$\rightarrow Q = \frac{-10n^2}{2} + 50n + c$$

$$Q(1) = 70 \rightarrow c = 25 \text{ لكن}$$

$$\rightarrow Q = -5n^2 + 50n + 25$$

النوع الرابع : مسائل حياتية

أمـثـلـة

① تتكاثر البكتيريا وفق المعادلة :

$\frac{dm}{dn} = \frac{1}{4}m$ لكل ساعة حيث (م) تدل على عدد البكتيريا إذا كانت $m = 200$ في بداية التكاثر , فما عدد البكتيريا بعد (8) ساعات

$$\frac{dm}{dn} = \frac{1}{4}m \rightarrow dm = \frac{1}{4}m\,dn$$

$$\int \frac{dm}{m} = \int \frac{1}{4}\,dn \rightarrow \ln m = \frac{1}{4}n + c$$

$$m = 200 \text{ عندما } n = 0$$

$$\ln 200 = 0 + c \rightarrow c = \ln 200$$

$$\ln m = \frac{1}{4}n + \ln 200$$

عند $n = 8 \rightarrow ln\, m = \frac{1}{4} \times 8 + \ln 200$

$$= 2 + \ln 200$$

---

② يزداد عدد سكان مدينة بمعدل (0.02) من عددهم سنويا , إذا كان عدد السكان الحالي (100000) , فما عددهم بعد (20) سنة

$$\frac{dw}{dn} = 0.02\, w \rightarrow \int \frac{dw}{w} = \int 0.02\, n$$

$$= ln\, w = 0.02n + c$$

لكن $ln\, 100000 = 0 + c \rightarrow c = ln\, 100000$

$$\ln w = 0.02\, n + \ln 100000$$

بعد (20) سنة

$$\ln w = 0.02 \times 20 + \ln 100000$$

---

③ تنمو البكتيريا بمعدل $\frac{dw}{dn} = \frac{3000}{1 + 0.25n}$

في الساعة , احسب عدد البكتيريا بعد مرور (4) ساعات , علمًا بأن العدد الأصلي هو (1000)

$$\int dw = \int \frac{3000}{1 + 0.25n} dn$$

$$w = \frac{3000}{0.25} \ln|1 + 0.25n| + c$$

لكن $w = 1000$ عندما $n = 0$

$$1000 = \frac{3000}{0.25} \ln|1 + 0| + c \rightarrow 1000$$

$$w = \frac{3000}{0.25} \ln|1 + 0| + c \rightarrow \quad c = 1000$$

عندما $n = 4$

$$w = \frac{3000}{0.25} \ln|1 + 0.25 \times 4| + 1000$$

$$= 12000 \ln 2 + 1000$$

أمثلة عامة

① حل المعادلات التفاضلية الآتية :

a) $(x^2 - 3x)\, dy = e^{-y}(x^2 + x - 12)\, dx$

$$\frac{dy}{e^{-y}} = \frac{x^2 + x - 12}{x^2 - 3x} dx$$

$$\frac{dy}{e^{-y}} = \frac{x^2 + x - 12}{x^2 - 3x} dx$$

$$e^y dy = \frac{(x + 4)(x - 3)}{x(x - 3)} dx$$

$$e^y dy = \left(1 + \frac{4}{x}\right) dx$$

$$\int e^y dy = \left(1 + \frac{4}{x}\right) dx$$

$$e^y\, dy = x + 4 \ln|x| + c$$

$$y = \ln(x + 4 \ln|x| + c)$$

---

b) $x^3\, dy - y\, dx = 0$

$$x^3\, dy = y\, dx \rightarrow \frac{dy}{y} = \frac{dx}{x^3}$$

$$\int \frac{dy}{y} = \int \frac{dx}{x^3}$$

$$\ln|y| = \frac{x^{-2}}{-2} + c$$

$$\ln|y| = \frac{-1}{2x^2} + c \rightarrow |y| = e^{-\frac{1}{2x^2}} + c$$

$$y = \int \frac{1}{4}\sin^2 x \; dx$$

$$y = \int \frac{1}{4} \times \frac{1}{2}(1 - cos\, 2x) \; dx$$

$$y = \frac{1}{8}\left(x - \frac{1}{2}\sin 2x\right) + c$$

---

f) $\frac{dy}{dx} = 1 - y + x^2 - yx^2$

$$\frac{dy}{dx} = (1 - y) + x^2(1 - y)$$

$$\frac{dy}{dx} = (1 - y)(1 + x^2)$$

$$\frac{dy}{1 - y} = (1 + x^2)dx$$

$$\int \frac{dy}{1 - y} = \int (1 + x^2)\, dx$$

$$-\ln|1 - y| = x + \frac{x^3}{3} + c$$

---

g) $(x^2 + 3x)\frac{dy}{dx} = e^{-2y}(x + 1)(x^2 - 9)$

$$\frac{dy}{e^{-2y}} = \frac{(x + 1)(x^2 - 9)}{x^2 + 3x}$$

$$e^{2y}dy = \frac{(x + 1)(x - 3)\cancel{(x + 3)}}{x\cancel{(x + 3)}}\, dx$$

$$\int e^{2y}\, dy = \int \frac{x^2 - 2x - 3}{x}\, dx$$

$$\frac{1}{2}e^{2y} = \int \frac{x^2}{x} - \frac{2x}{x} - \frac{3}{x}\, dx$$

$$\frac{1}{2}e^{2y} = \int x - 2 - \frac{3}{x}\, dx$$

c) $dx - 3\, dy = \cos x \; dx$

$$dx - \cos x \; dx = 3\, dy$$

$$(1 - \cos x)\, dx = 3\, dy$$

$$\int (1 - \cos x)\, dx = \int 3 \; dy$$

$$x - \sin x + c = 3y$$

$$y = \frac{1}{3}(x - \sin x + c)$$

---

d) $e^{-y}\sin x - \frac{dy}{dx}\cos^2 x = 0$

$$e^{-y}\sin x = \frac{dy}{dx}\cos^2 x$$

$$\frac{sinx\, dx}{cos^2 x} = \frac{dy}{e^{-y}}$$

$$\int \sec x \tan x\, dx = \int e^y \; dy$$

$$\sec x + c = e^y \rightarrow y = \ln|\sec x| + c$$

---

e) $\sec^2\frac{x}{2}\, dy - \sin^2\frac{x}{2}\, dx = 0$

$$\sec^2\frac{x}{2}\, dy = \sin^2\frac{x}{2}\, dx$$

$$dy = \frac{\sin^2\frac{x}{2}}{\sec^2\frac{x}{2}}\, dx$$

$$dy = \sin^2\frac{x}{2}\cos^2\frac{x}{2}\, dx$$

$$dy = \left(\sin\frac{x}{2}\cos\frac{x}{2}\right)^2$$

$$\int dy = \int \left(\frac{1}{2}\sin x\right)^2 \; dx$$

$$\frac{1}{2}e^{2y} = \frac{x^2}{2} - 2x - 3\ln|x| + c$$

$$e^{2y} = 2\left(\frac{x^2}{2} - 2x - 3\,ln|x| + c\right)$$

$$2y = \ln\left(2\left(\frac{x^2}{2} - 2x - 3\,ln|x| + c\right)\right)$$

$$y = \frac{1}{2}\left(ln\left(2\left(\frac{x^2}{2} - 2x - 3\,ln|x| + c\right)\right)\right)$$

---

② يتحرك جسيم حسب العلاقة : $t = \sqrt{w}$
وكانت السرعة بعد بدء الحركة تساوي $(9)m/sec$ وقطع مسافة $(80)m$ خلال **(4)** ثواني , فما المسافة التي قطعها الجسيم بعد ثانيتين من بدء الحركة

$$t = \frac{dw}{dn} = \sqrt{w} \rightarrow \frac{dw}{\sqrt{w}} = dn$$

$$\int w^{-\frac{1}{2}}\,dw = \int dn \rightarrow \frac{2w^{\frac{1}{2}}}{1} = n + c$$

$$w = 9 \quad , \quad n = 0$$

$$2(9)^{\frac{1}{2}} = 0 + c \rightarrow 2 \times 3 = c$$

$$2w^{\frac{1}{2}} = n + 6$$

$$w^{\frac{1}{2}} = \frac{1}{2}(n+6) \rightarrow w = \left(\frac{1}{2}(n+6)\right)^2$$

$$\frac{df}{dn} = \frac{1}{4}(n+6)^2 \rightarrow df = \frac{1}{4}(n+6)^2\,dn$$

$$\int df = \int \frac{1}{4}(n+6)^2\,dn$$

$$f = \frac{1}{4} \times \frac{(n+6)^3}{3} + c$$

$n = 4$ عند $f = 80$

$$80 = \frac{(4+6)^3}{12} + c$$

$$80 = \frac{1000}{12} + c \rightarrow c = \frac{-40}{12}$$

$$f = \frac{(n+6)^2}{12} - \frac{40}{12}$$

$$f(2) = \frac{(2+6)^2}{12} - \frac{40}{12} = \frac{512}{12} - \frac{40}{12}$$
$$= \frac{512}{12} - \frac{40}{12} = \frac{472}{12}$$

---

③ قذفت كرة من برج ارتفاعه $(45)m$ عن سطح الأرض إلى أعلى وبسرعة ابتدائية $40m/sec$ وبتسارع $-10m/sec^2$, فما الزمن الذي استغرقته الكرة لتعود للأرض

$$t = \frac{dw}{dn} = -10 \rightarrow dw = -10\,dn$$

$$\int dw = \int -10\,dn \rightarrow w = -10n + c$$

$$w = 40 \quad , \quad n = 0$$

$$40 = 0 + c \rightarrow c = 40$$

$$w = -10n + 40$$

$$\frac{df}{dn} = (-10n + 40)\,dn$$

$$\int df = \int (-10n + 40)\,dn$$

$$f = -5n^2 + 40n + c$$

$$f(0) = 0 + 0 + c = 45$$

$$f(0) = -5n^2 + 40n + 45 = 0 \quad \div (-5)$$

يصل الأرض عندما $f = 0$

$-5n^2 + 40n + 45 = 0 \qquad \div (-5)$

$n^2 - 8n - 9 = 0 \rightarrow (n-9)(n+1) = 0$

$n = 9, -1 \qquad \therefore n = 9$

---

④ إذا كان ميل المماس لمنحنى (y) عند (x, y) هو $\frac{e^{x-y}}{e^x + 1}$, فما قاعدة الاقتران المار بالنقطة $(1, 0)$

$\frac{dy}{dx} = \frac{e^{x-y}}{e^x + 1} \rightarrow \frac{dy}{dx} = \frac{\frac{e^x}{e^y}}{e^x + 1}$

$\frac{dy}{dx} = \frac{e^x}{e^y(e^x + 1)}$

$\int e^y = \int \frac{e^x}{e^x + 1} dx$

$e^y = \ln|e^x + 1| + c$

$(1, 0) \rightarrow e^0 = \ln|e + 1| + c$

$1 = \ln|e + 1| + c$

$e^y = \ln|e^x + 1| + (1 - ln|e^1 + 1|)$

$y = \ln(ln|e^x + 1|) + (1 - \ln|e + 1|)$

⑤ يمثل الشكل العلاقة بين السرعة والزمن , احسب المسافة على $[0, 7]$

$f = \int w \ dn$

$=$ المساحة تحت المنحنى

$= \frac{1}{2} \times$ مجموع القاعدتين $\times$ الارتفاع

$= \frac{1}{2} \times (7 + (6 - 2)) \times 30$

$= \frac{1}{2} \times 11 \times 30 = 165$