Jordan University of Science & Technology
Department of Electrical Engineering

Fall Semester 2010/2011
Second Examination

EE447	Digital Control	12.12.2010
Name:	University # :	

Q.1 Consider a C/L sampled-data system with $G_p(s) = 1/s(s+1)$, predeeded by a Z.O.H. and a sampling intervlal of T sec. Find:

a) The location of O/L poles on the Z-Plane When T = 1 sec.
b) it can be shown that for T = 1, ξ (Zeta) = 0.25 and τ = 4.36 sec. Calculate ξ (Zeta) and τ when T = 0 sec (continuous that is), 0.2 sec. Discuss effect of T on system Parameters. Hence find Percent Overshoot and Settling Time T_s.

Q.2 Consider the control system shown in Fig Q.2. The System goals are to control the effect of disturbance and steady state error due to a Ramp input.

a) Let $D(z) = K_p$, find the steady state error due to a ramp input, and the $C_{s.s.}$ due to step disturbance.
b) Repeat the calculation in (a) with $D(z) = K_p + K_I\, Tz/(z-1)$. Discuss the effect of adding such controller.

Q.3 Consider a Sampled data system with Gp(s) = 0.8/(3s+1), preceeded by a Z.O.H. and a sampling interval T.

a) Find the range of sampling time T for stability; using Jury's method.
b) using D(z) = K, T = 0.5. Plot the Root Locus in Z-Plane, Find the range of gain K for stable system from RL Plot.
c) To imptove system performance a Lead Compensator is added, with a TF:
$G_C(s) = K_C\,(s+a)/(s+b)$ with a = 0.5, b = 5
Plot the Root Locus and find the gain K_C so that the desired pole is at s = -8.9.
Find D(z) using MPZ method with T = 0.5 sec.
d) Plot the Root Locus for the system with the controller found in (c).

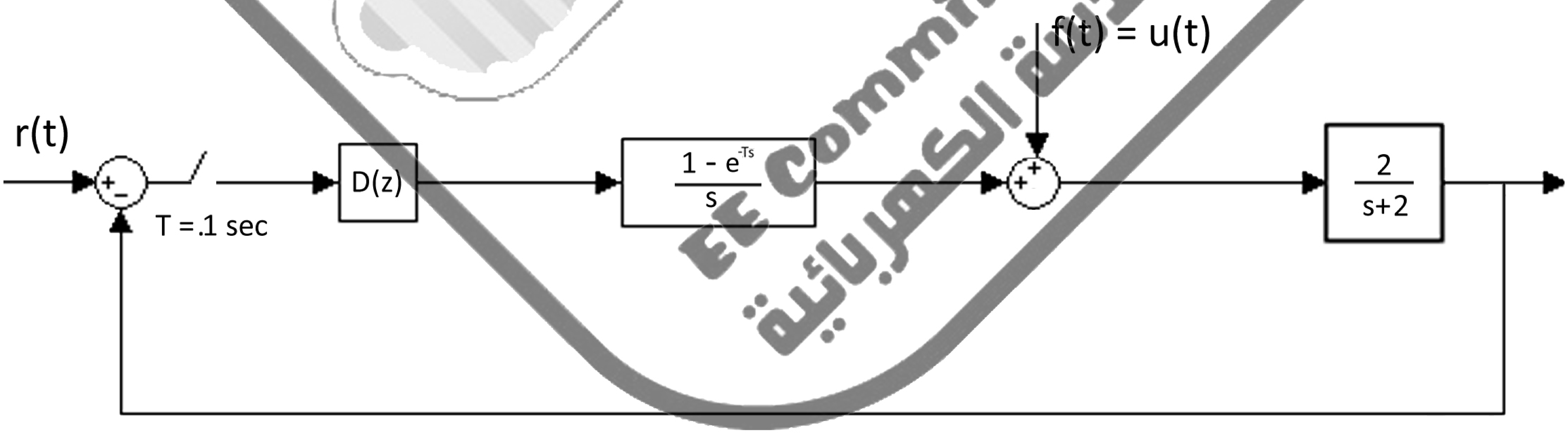


Fig Q.2.

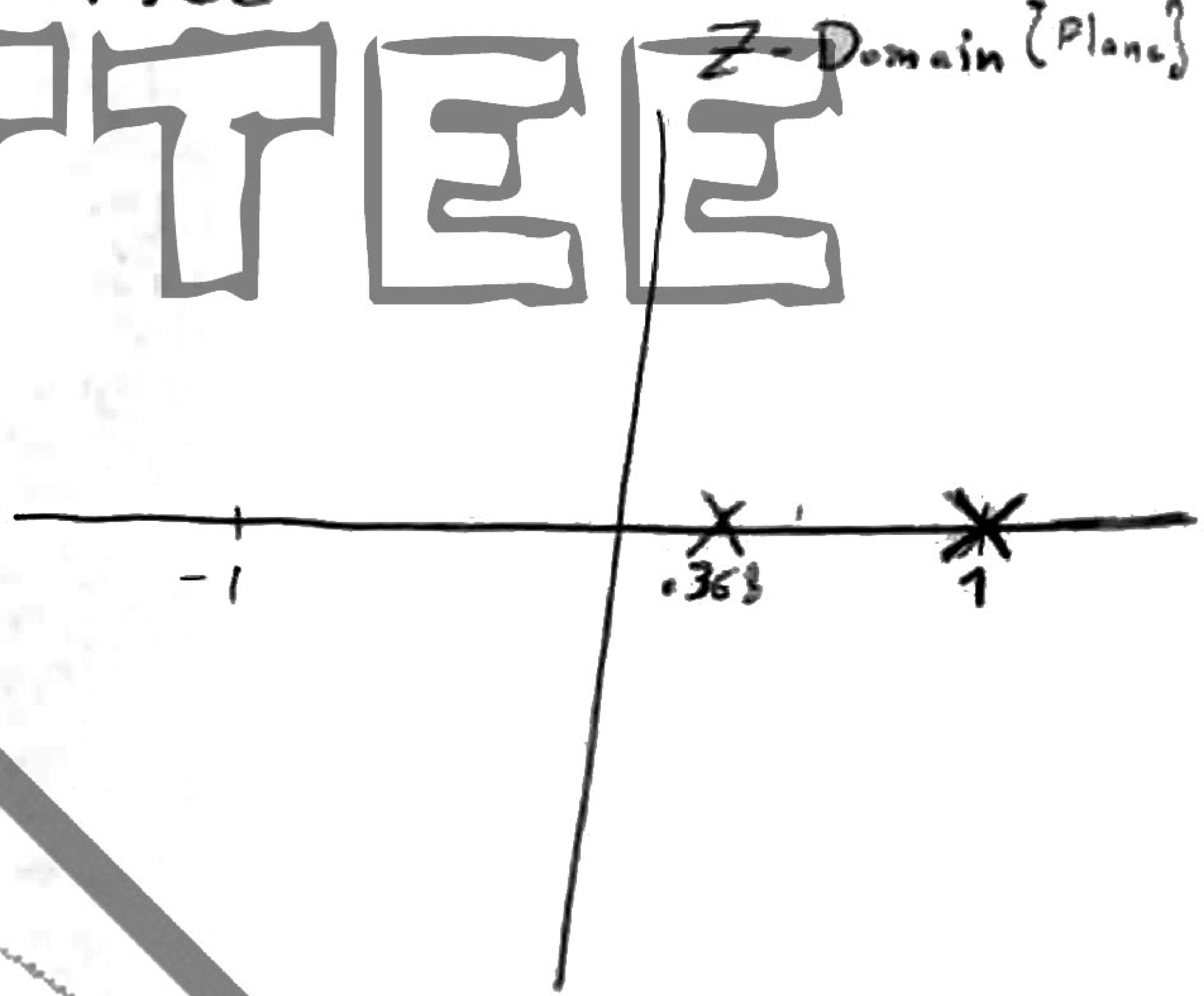
Digital Control 2nd Exam {Post-Exam Review}

Q1 a) $G_p(s) = \dfrac{1}{s(s+1)}$

O/L poles of $G_p(s)$ on Z-plane, $T = 1\,sec$

$Z = e^{sT} \Rightarrow Z_1 = e^{0 \times 1} = 1$

$Z_2 = e^{-1 \times 1} = .368$



b) $G(z) = \underbrace{\left[\dfrac{Z-1}{Z}\right]}_{Z.O.H} * \underbrace{Z\left\{\dfrac{1}{s^2(s+1)}\right\}}_{G_p(s)/s}$

from Tables $\Rightarrow \dfrac{1}{s^2(s+1)} \rightleftharpoons \dfrac{Z\left[(T-1+e^{-T})z + (1-e^{-T}-Te^{-T})\right]}{(Z-1)^2(Z-e^{-T})}$

$\therefore G(z) = \dfrac{(T-1+e^{-T})z + (1-e^{-T}-Te^{-T})}{(Z-1)(z-e^{-T})}$ ..... $\circledast$

① for $T = 1$, $\xi = 0.25$, $\tau = 4.36$ second {given}

$P.O. = 100\, e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}} = 44.43\%$

$T_s = 4\tau = 17.44s$

② for $T = .2$, Substitute $T = .2$ in $\circledast$

$G(z) = \dfrac{.01873\,z + .017523}{(Z-1)(Z-0.81873)}$

Char. eqn $\{1 + G(z)\}$ gives: $Z^2 - 1.8\,z + .836253 = 0$

hence:

$$Z = .9 \mp j .1620277754$$

$$Z = \underbrace{0.9145}_{r} \angle \underbrace{\pm .178}_{\theta}$$

$$\xi = \frac{-\ln(r)}{\sqrt{\ln(r)^2 + \theta^2}} = .4487, \quad P.O. = 20.65\%$$

$$\tau = \frac{1}{\xi \omega_n} = \frac{1}{\ln(r)} = 2.2377 s, \quad T_s = 4\tau = 8.95 s$$

③ for $T = 0$, {continuous that is}:

$$G(s) = \frac{1}{s(s+1)}$$

$1 + G(s) = 0$, gives $s^2 + s + 1$

$$\omega_n = 1, \quad \xi = \frac{1}{2\omega_n} = \frac{1}{2}$$

$$P.O. = 16.30\%$$

$$\tau = \frac{1}{\xi \omega_n} = 2 s$$

$$T_s = 4\tau = 8 s$$

Q2

a) $D(z) = k_p$, $r(t) = t$

$$R(z) = \frac{Tz}{z-1}$$

1) e_{ss} due to Ramp input can be directly calculated using:

$$e_{ss} = \frac{1}{K_v} \quad , \text{ where } K_v = \lim_{z\to 1} \frac{1}{T}(z-1)\,G(z)\,D(z)$$

$$G(z) = \underbrace{\left\{\frac{z-1}{z}\right\}}_{Z.O.H} \underbrace{Z\left\{\frac{2}{s(s+2)}\right\}}_{Gp(s)/s}$$

from Tables $\dfrac{1}{s(s+2)} \rightleftharpoons \dfrac{z(1-e^{-2T})}{(z-1)(z-e^{-2T})}$, $T = .1\,s$

$$G(z) = \frac{.18127}{(z-.81873)} \quad , \quad D(z)G(z) = \frac{.18127\,k_p}{(z-.81873)}$$

$$\therefore k_v = \lim_{z\to 1} \frac{1}{.2}(z-1) * \frac{.18127\,k_p}{(z-.81873)} = \text{Zero}$$

$\therefore$ e_{ss} = infinity $\{\infty\}$

2) $C(z) = \dfrac{\overline{GU}(z)}{1 + D(z)G(z)}$, $U(z)$ = input disturbance

$D(z) = k_p$, $DG(z) = \dfrac{.18127\,k_p}{(z-.81873)}$, $\overline{GU}(z) = \dfrac{.18127\,z}{(z-1)(z-.81873)}$

$$\therefore C(z) = \frac{.18127\,z}{(z-1)\left[(z-.81873) + .18127\,k_p\right]}$$

$$C_{ss} = \lim_{z\to 1}(z-1) * C(z) = \lim_{z\to 1} \frac{.18127\,z}{(z-.81873) + .18127}$$

$$C_{ss} = \frac{.18127}{.18127 + .18127\,k_p} = \frac{1}{1+k_p}$$

b) 1. $D(z) = k_p + \frac{k_I * .1 z}{z-1}$

$$= \frac{k_p z - k_p + .1 k_I z}{z-1} = \frac{(k_p + .1k_I) z - k_p}{z-1}$$

$$= \frac{K(z-\alpha)}{(z-1)} \quad , \text{ where: } K = k_p + .1k_I$$

$$\alpha = \frac{k_p}{k_p + .1k_I}$$

as in previous section:

$$e_{ss} = \frac{1}{K_v} \quad , K_v = \lim_{z\to 1} \frac{1}{T}(z-1)\, G(z)\, D(z)$$

$$K_v = \lim_{z \to 1} \frac{1}{.1} * \cancel{(z-1)} \frac{.18127}{(z-.81873)} * \frac{K(z-\alpha)}{\cancel{(z-1)}}$$

$$= 10 * \frac{\cancel{.18127}}{\cancel{.18127}} * \frac{\cancel{k_p} + .1k_I - \cancel{k_p}}{1} = k_I$$

$\therefore$ $e_{ss} = \frac{1}{k_I}$ {Much Less than infinity, system is mor stable}

2. $C(z) = \frac{\overline{GU}(z)}{1 + D(z)G(z)}$, $\overline{GU}(z) = \frac{.18127 z}{(z-1)(z-.81873)}$

$D(z)G(z) = \frac{.18127 K(z-\alpha)}{(z-1)(z-.81873)}$ U is from disturbance !

$$C(z) = \frac{.18127 z}{(z-1)(z-.81873) + (z-\alpha).18127K}$$

$$C_{ss} = \lim_{z\to 1} (z-1)\, C(z) = \lim_{z\to 1} \frac{.18127 z}{(z-.81873) + \frac{.18127K(z-\alpha)}{(z-1)}}$$

$$C_{ss} = \frac{.18127}{.18127 + \underbrace{\frac{.18127K(1-\alpha)}{}}_{\text{Zero}}} = \frac{1}{\infty} = \text{Zero} !$$

{Steady state value of Output due to Disturbance equals to Zero, which is better than previous section}

Q3 a) $G_p(s) = \frac{.8}{3s+1} = .8 * \frac{1/3}{s+1/3}$

$$G(z) = \left[\frac{z-1}{z}\right] * .8\, \mathcal{Z}\left\{\frac{1/3}{s(s+1/3)}\right\}$$

from Tables: $\frac{1/3}{s(s+1/3)} \Rightarrow \frac{z(1-e^{-T/3})}{(z-1)(z-e^{-T/3})}$

$$G(z) = \frac{.8(1-e^{-T/3})}{(z-e^{-T/3})}$$

$1+G(z) = Q(z)$ {char eqn}

$$z - e^{-T/3} + .8 - .8e^{-T/3} = Q(z)$$

$$\therefore Q(z) = \underbrace{z}_{a_1} \underbrace{-1.8e^{-T/3}+.8}_{a_0}$$

1) $Q(1) > 0$

$1-1.8e^{-T/3}+.8>0 \Rightarrow \frac{1.8}{1.8} > e^{-T/3}$, $\frac{-T}{3} < \ln(1)$

$\therefore T > 0 \ldots 1$

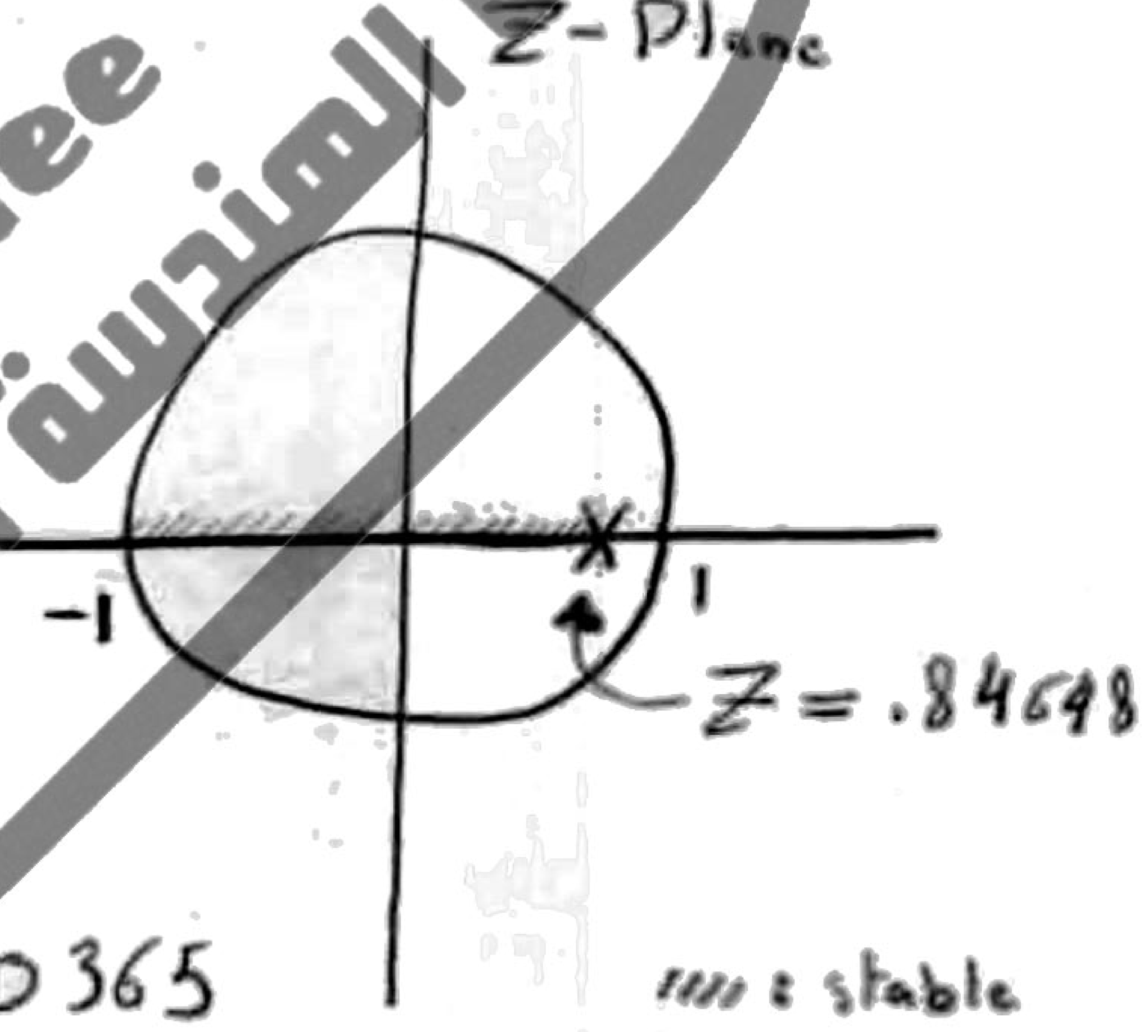
2) $(-1)^1 Q(-1) > 0$

$1+1.8e^{-T/3} - .8 > 0 \Rightarrow 1.8e^{-T/3} > .2$

$-T/3 > \ln\left(\frac{.2}{1.8}\right) \Rightarrow T < 6.59167$ $\quad\therefore 6.59167 > T > 0$

b) $D(z) = K$, $T = \frac{1}{2}$, $G(z) = \frac{.1228}{z-.84648}$

$$D(z)G(z) = \frac{.1228K}{z-.84648}$$



for stable system K at $z=-1$

$$\therefore K = \left|\frac{z-.84648}{.1228}\right|_{z=-1} = 15.0365$$

{checked by Matlab}

$$\therefore 0 < K < 15.0365$$

c) $G_c(s) = K_c \dfrac{(s+a)}{(s+b)}$, $a = \frac{1}{2}$, $b = 5$

$G_c(s) = K_c \dfrac{(s+.5)}{(s+5)}$, desired Pole $s = -8.9$

$G_c G_p(s) = \dfrac{K_c * .8 * \frac{1}{3} (s+.5)}{(s+5)(s+\frac{1}{3})}$, to find K_c at $s = -8.9$

$$K_c = \left| \frac{(s+5)(s+\frac{1}{3})}{.8 * \frac{1}{3} * (s+.5)} \right|_{s=-8.9} = 14.9152$$

for MPZ: ① $Z = e^{sT}$

$$\therefore D(z) = K_D \frac{Z - e^{-\frac{1}{4}}}{Z - e^{-\frac{5}{2}}} = K_D \frac{(Z-.7788)}{(Z-.0821)}$$

② No. of Zeros = No. of Poles , then no need to add any $(Z+1)$ in the numerator.

③ $\lim_{s \to 0} D(s) = \lim_{z \to 1} D(z)$, in order to find K_D

$$\therefore 14.9152 * \frac{.5}{5} * 1 = K_D * \frac{(1-.7788)}{(1-.0821)} \Rightarrow K_D = 6.189$$

$$\Rightarrow D(z) G(z) = \frac{.76 (Z-.7788)}{(Z-.84648)(Z-.0821)}$$ $\{6.189 * .1228 = .76\}$

d) Root Locus Plot

{checked by M.Hab}