Second

Mon . 31th . oct . 2016

## ① Continuity. الاتصال.

* Defined: A function (f) is said to be a continuos at $x=c$, if the following conditions are satisfied:

[1] $f(c)$ is defined

[2] $\lim_{x\to c} f(x)$ exist.

[3] $f(c) = \lim_{x\to c} f(x)$.

Ex:- Determine if the following function are continouse at $x=2$.

① $f(x) = \dfrac{x^2-4}{x-2}$

② $f(x) = \begin{cases} x^2-4 & \text{if } x \neq 2 \\ 3 & \text{if } x = 2. \end{cases}$

③ $f(x) = \begin{cases} \dfrac{x^2-4}{x-2} & \text{if } x \neq 2 \\ 4 & \text{if } x = 2. \end{cases}$

Home-work ④ $f(x) = \begin{cases} \dfrac{|x-2|}{x-2} & \text{if } x \neq 2 \\ 1 & \text{if } x = 2. \end{cases}$

Sol(2)  1. $f(2) = 3.$

2. $\lim_{x\to 2} = f(x) = 4$

$3 \neq 4$

$\Rightarrow f(2) \neq \cancel{\neq} \lim_{x\to 2} f(x)$

So $f(x)$ is dis continous at $x = 2$

Sol (1)

$f(x) = x + 2; \; x \neq 2$

$f(2)$ is undefined

So $f(x)$ is dis continous at $x = 2.$

Ex:- find the value of the (a and b) so that $f(x)$ is continous :-

$$f(x) = \begin{cases} 3x^2 + ax & \text{if } x \leq 1 \\ \frac{x^2 - 4}{x - 2} & \text{if } 1 < x < 2 \\ 4b + 2x & \text{if } x \geq 2 \end{cases}$$

Sol $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^-} f(x)$

$\frac{-3}{-1} = 3 + a \qquad \boxed{a = 0}$

ت.م to find the value of b.

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$$

$$4 = 4b + 4 \longrightarrow \boxed{b=0}$$

$\boxed{\text{Wed. 2}^{\text{nd}}\text{. Nov. 2016}}$

* Derivative . اشتقاق

- Define : A function ($f$) is said to be different all at $x$ if :-
!!

$$f(x) = \lim_{h \to 0} \frac{f(x)+(h) - f(x)}{h} \text{ exists.}$$

~~No tation~~

Notation: $y = f(x)$

$$\frac{dy}{dx} = f'(x).$$

Ex:- Use the definition of derivation to find the following :

① $f(x) = x^2 + 4x$

② $f(x) = \frac{1}{\sqrt{x}}$.

sol $\longrightarrow$

Sol(i) $f(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$$= \lim_{h \to 0} \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h} = \frac{0}{0}$$

$$= \lim_{h \to 0} \frac{\sqrt{x} - \sqrt{x+h}}{h(\sqrt{x})(\sqrt{x+h})}$$

$$\cancel{= \lim_{h \to 0} \frac{\sqrt{x} - \sqrt{x+h}}{h(\sqrt{x})(\sqrt{x+h})} * \frac{\sqrt{x} + \sqrt{x+h}}{\sqrt{x} +}}$$

$$= \lim_{h \to 0} \frac{\sqrt{x} - \sqrt{x+h}}{h(\sqrt{x})(\sqrt{x+h})} * \frac{\sqrt{x} + \sqrt{x+h}}{\sqrt{x} + \sqrt{x+h}}$$

$$f(x) = \lim_{h \to 0} \frac{\cancel{x} - \cancel{x} - \cancel{h}}{\cancel{h}(\sqrt{x})(\sqrt{x+h})(\sqrt{x} + \sqrt{x+h})}$$

$$= \lim_{h \to 0} \frac{-1}{(\sqrt{x})(\sqrt{x+h})(\sqrt{x} + \sqrt{x+h})}$$

$$= \frac{-1}{(\sqrt{x})(\sqrt{x})(\sqrt{x} + \sqrt{x})}$$

$$= \frac{-1}{x(2\sqrt{x})} = \frac{-1}{2x^{3/2}}$$

$$f(x) = \frac{-1}{2\sqrt{x^3}}$$

# * Differentiation Rules قواعد الاشتقاق

| function | Derivative. |
| --- | --- |
| ① $f(x) = k$ | $f'(x) = 0$ |
| ② $f(x) = x^n$ | $f'(x) = n\,x^{n-1}$ |
| ③ $f(x) = g(x)\,h(x)$ | $f'(x) = g(x)k(x) + h(x)\,g(x)$ |
| ④ $f(x) = \frac{g(x)}{h(x)}$ | $f'(x) = \frac{h(x)\,g(x) - g(x)\,h(x)}{h(x)^2}$ |
| ⑤ $f(x) = \frac{h(x)}{\sqrt{g(x)}}$ | $f'(x) = \frac{g'(x)}{2\sqrt{g(x)}}$ |
| ⑥ $f(x) = \frac{a}{g(x)}$ | $f'(x) = \frac{-a\;g'(x)}{(g(x))^2}$ |
| ⑦ $f(x) = g(x) \mp h(x)$ | $f'(x) = g'(x) \mp h(x)$ |
| ⑧ $f(x) = a\,g(x)$ | $f'(x) = a\,g'(x)$ |
| ⑨ $f(x) = (g(x))^n$ | $f'(x) = n(g(x))^{n-1} * g'(x)$ |
| ⑩ $y = f(x)$, $u = f(x)$ | $\frac{dy}{dx} = \frac{dy}{dx} + \frac{du}{dx}$ |
| ⑪ $y = f(x)$, $u = f(s)$, $s = f(x)$ | $\frac{dy}{dx} = \frac{dy}{du} + \frac{du}{ds} + \frac{ds}{dx}$. |

**Ex:-** Find $f'(x)$ or $\dfrac{dy}{dx}$ for :-

① $f(x) = \sqrt{x^5 + 4x}$

② $y = x^5 + \dfrac{1}{\sqrt{x^3}} + 5$

③ $f(x) = \dfrac{3}{(x^2 + 4x)}$

④ $f(x) = (x^5 + 4x^2 + 3x)^7$

⑤ $f(x) = \dfrac{x^2 + 4}{3x^5 + 4x}$

⑦ $f(x) = \dfrac{1}{\sqrt[3]{(x^4 + 2)^3}} + 5x$

⑥ $f(x) = (\sqrt{x^2 + 4})(3x^3 + 4)$

⑧ $y = U^3 + 4U$
$U = \sqrt{3x + 6}$

**Sol(1)** $f'(x) = \dfrac{5x^4 + 4}{2\sqrt{x^5 + 4x}}$

**Sol(2)** $y = x^5 + x^{-\frac{3}{2}} + 5$

$$\frac{dy}{dx} = 5x^4 \frac{-3}{2} x^{\frac{-5}{2}} + 0$$

$$= 5x^4 - \frac{3}{2\sqrt{x^5}}$$

Sol(3) $f(x) = \frac{-3(2x+4)}{(x^2+4x)^2}$

Sol(4.) $f(x) = 7(x^5+4x^2+3x)^6 *$
$(5x^4+8x+3)$

Sol(5). $f(x) = \frac{(3x^5+4x)*2x - [(x^2+4)(15x^4+4)]}{}$

Sol(6) $f(x) = (\sqrt{x^2+4})(9x^2) + (3x^3+4)\left(\frac{2x}{2\sqrt{x^2+4}}\right)$

Sol(7). $f(x) = ((x^2+2)^3+5x)^{\frac{-1}{3}}$

$f(x) = \frac{-1}{3}\left((x^4+2)^3+5x\right)^{\frac{-4}{3}} * \left[3(x^4+2)^2 * 4x^3+5\right]$

Sol(8). we solve it using the
"Chain Rule." قاعدة السلسلة

$$\frac{dy}{dx} = \frac{dy}{du} * \frac{du}{dx}$$

$$= (3u^2+4)\left[\frac{3}{2\sqrt{3x+6}}\right]$$

$$= \left[3(\sqrt{3x+6})^2+4)\right]\left[\frac{3}{2\sqrt{3x+6}}\right]$$

تابع (8)

$$= \left[(9x + 22)\right]\left[\frac{3}{2\sqrt{3x+6}}\right]$$

Ex:- let $f(x) = \sqrt{x^2+5}$ and

سؤال مش دقيق

$g(x) = \frac{1}{x^5+4x}$ find

$(f\circ g)'(x)$ , $(f\circ g)'(2)$ , $(f\circ g)(2)$

$\Rightarrow$ zero

Sol $(f\circ g)(x) = f(g(x))$

$(f\circ g)'(x) = f'(g(x)) * g'(x)$

but $f'(x) = \frac{2x}{2\sqrt{x^2+5}} = \frac{x}{\sqrt{x^2+5}}$

$$g'(x) = \frac{-(5x^4+4)}{(x^5+4x)^2}$$

but $(f\circ g)'(x) = f'(g(x)) * g'(x)$

$$= \frac{f\left(\frac{1}{x^5+4x}\right)}{\sqrt{\left(\frac{1}{x^5+4x}\right)^2+5}} * \left[\frac{-(5x^4+4)}{(x^5+4x)^2}\right]$$

$$(f\circ g)'(1) = \frac{\frac{1}{5}}{\sqrt{\frac{1}{25}+5}} * \frac{-9}{25}$$

$\Longrightarrow$

* Equation of the tangent line معادلة المماس

The equation of the tangent line to ~~y=~~ $y = f(x)$ at $(x_0, y_0)$

is $y - y_0 = m(x - x_0)$, where

Slope $= m = \dfrac{dy}{dx} = f'(x_0)$

Ex: Find the slope and the equation of the tangent line to $f(x) = \sqrt{x+5}$ at $x = 2$?

Sol slope $= m = f'(2) = \left.\dfrac{1}{2\sqrt{x+5}}\right|_{x=4}$

$$= \frac{1}{2\sqrt{4+5}}$$

$$= \frac{1}{6}$$

$\boxed{x_0 = 4}$

$y_0 = \sqrt{4+5} = \boxed{3}$

Equation : $y - y_0 = m(x - x_0)$

$y - 3 = \frac{1}{6}(x - 4)$

Ex 9:

$$y - 3 = \frac{1}{6}(x - 4)$$

$$6y - 18 = x - 4$$

$$6y - x - 14 = 0.$$

Mon. $7^{th}$ Nov. 2016

* Higher derivation المشتقات العليا

* IF $y = f(x)$, then

$$\frac{dy}{dx} = f'(x)$$

$$\frac{d^2y}{dx^2} = f''(x)$$

$$\frac{d^3y}{dx^3} = f'''(x)$$

$$\frac{d^4y}{dx^4} = f^{(4)}(x)$$

Ex Find $\left.\frac{d^3y}{dx^3}\right|_{x=2} = f'''(2)$ for

① $y = \dfrac{x}{x+1}$

② $f(x) = \dfrac{1}{x}$

Sol(2) $f'(x) = \dfrac{(-1)(1)}{x^2} = \dfrac{-1}{x^2}$

$$f''(x) = \frac{-(-1) * 2x}{x^4} = \frac{2}{x^3}$$

$$f'''(x) = \frac{-(2) * 3x^2}{x^6} = \frac{-6}{x^4}$$

$$f''' = \frac{-6}{(2)^4} = \frac{-6}{16} = \frac{-3}{8}$$

* L'Hôpital Rule قاعدة لوبتال

**Thm** let $f(x)$ and $g(x)$ be continuous and differentiable function at $x=a$ and $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{0}{0}$ Then $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$

**Ex:-** Use L'Hôpital Rule to find

① $\lim_{x \to 1} \frac{x^8 - 1}{x^2 - x} = \frac{0}{0}$

② $\lim_{h \to 0} \frac{(1+2h)^{20} - 1}{h} = \frac{0}{0}$

Solution (1) $\Longrightarrow$

Sol (1) $\lim_{x\to 1} \dfrac{x^8-1}{x^2-x}$

$= \lim_{x\to 1} \dfrac{8x^7}{2x-1} = \boxed{8}$

Sol (2) $\lim_{h\to 0} \dfrac{(1+2h)^{20}-1}{h}$

$= \lim_{h\to 0} \dfrac{20(1+2h)^{19} * 2 - 0}{1}$

$= 20(1)^{19}(2) = \boxed{40}$

## * Derivative of the trigonometric functions

الاقترانات الدائرية (المثلثية).

| Function | Derivative. |
| --- | --- |
| جا س * $f(x) = \sin x$ | $f'(x) = \cos x$ |
| جتا س * $f(x) = \cos x$ | $f'(x) = -\sin x$ |
| ظا س * $f(x) = \tan x$ | $f'(x) = \sec^2 x$ |
| ظتا س * $f(x) = \cot x$ | $f'(x) = -\csc^2 x$ قتا س |
| قا س * $f(x) = \sec x$ | $f'(x) = \sec x \tan x$ |
| * $f(x) = \csc x$ | $f'(x) = -\csc x \cot x$ |

الأسية                                   اللوغرثمية

# * Derivative of Exponential and logarithmic functions

| |
|---|
| ن(س) = لو$_{ب}$ س |
| $f(x) = \log_b x$ |

| |
|---|
| ن(س) = لو$_{هـ}$ س |
| $f(x) = \log_e x$ |
| $= \ln x$ |
| logarthim natural |

| Function | Derivative |
|---|---|
| * $f(x) = e^x$ | $f'(x) = e^x$ |
| * $f(x) = b^x$ ← b = base | $f'(x) = b^x * \ln b$ |
| * $f(x) = \ln x$ | $f'(x) = \frac{1}{x}$ |
| * $f(x) = \log_b x$ | $f'(x) = \left(\frac{1}{x}\right) * \left(\frac{1}{\ln b}\right)$ |
| * $f(x) = \ln g(x)$ | $f'(x) = \frac{g'(x)}{g(x)}$ |

Ex:- find $f'(x)$ for:

① $f(x) = (\cos x)(x^2 + 4x)$

② $f(x) = \sec^3 x$

③ $f(x) = \tan(\sin x)$

④ ..... ⑨ →

(4) $f(x) = 3^x$

(5) $f(x) = e^{\sin x}$

(6) $f(x) = \ln (x^2 + 4x)$

(7) $f(x) = \log_3 \sin x$

(8) $f(x) = \ln (\cot x)$

(9) $f(x) = (\cos x)(x^2 + 4)$

Sol(1) $f'(x) = (\cos x) \quad (2x+4) + (x^2 + 4x)(-\sin x)$

Sol(2) $f'(x) = 3 \sec^2 x * (\sec x \tan x)$
$= 3 \sec^3 x \tan x.$

Sol(3) $f'(x) = \sec^2 (\sin x) * [\cos x)$

Sol(4) $f'(x) = 3^x * \ln 3$

Sol(5) $f'(x) = e^{\sin x} * [\cos x]$

Sol(6) $f'(x) = \dfrac{2x+4}{x^2+4x}$

sol(7) $f'(x) = \left(\dfrac{\cos x}{\sin x}\right) * \dfrac{1}{\ln 3}$

Sol(8) $f'(x) = \dfrac{-\csc^2 x}{\cot x}.$

Sol(9) $f'(x) = (\csc x)(2x) + (x^2+4)(-\csc x \cdot \cot x)$

* دائماً إلي تبدأ بحرف C $\leftarrow$ مشتقتها سالبة

* Previous Exams (Multiple choice)

① let $f(x) = x^2 - x - 1$ find $(f \circ f)'(1)$?

Ⓐ $-5$ Ⓑ $-3$ Ⓒ $5$ $\boxed{Ⓓ\ -15}$

Sol(1) Step(1) : $(f \circ f)(x) = f(f(x))$

Step(2) :- $(f \circ f)'(x) = f'(f(x)) * f'(x)$

Step(3) :- $(f \circ f)'(1) = f'(f(1)) * f'(1)$ $\boxed{*}$

$\Longrightarrow f'(x) = 4x^3 - 1 \Rightarrow f'(1) = 4(1)^3 - 1$

$= 3$

$f(x) = x^4 - x - 1 \Longrightarrow f(1) = (1)^4 - 1 - 1 = -1$

go back $\boxed{*}$ $(f \circ f)'(1) = f'(f(1)) * f'(1)$

$= f'(-1) * 3$

$= (4(-1)^3 - 1) * 3$

$= -5 * 3$

$= -15$

② Let $f(x) = (3x-2)^{20}$ Then $\lim_{h \to 0} \frac{f(1+h) - f(1)}{h^2 + 2h}$

$= \frac{0}{0}$

Ⓐ 10 Ⓑ 20 $\boxed{\text{Ⓒ } 30}$ Ⓓ 40 Ⓔ 50.

Sol (2) $\lim_{h \to 0} \frac{f'(1+h) * 1 - 0}{2h + 2} = \frac{f'(1)}{2} = \frac{60}{2} = 30$

but $f'(x) = 20(3x-2)^{19} * 3$

$\Longrightarrow f'(1) = 20(1)^{19} * 3 = 60$

* * * * *

③ let $f(x) = (g(x) + 3) \ln(x+1)$, $g(0) = 2$

find $f'(0)$

Ⓐ -1 $\boxed{\text{Ⓑ } 5}$ Ⓒ 0 Ⓓ 6

Sol (3) $f'(x) = (g(x) + 3) * \frac{1}{x+1} + \ln(x+1) * [g'(x) + 0]$

$\longrightarrow$

$$\acute{f}(0) = (g(0)+3) * \frac{1}{0+1} + \ln(0+1) * [\acute{g}(0)]$$

$$\boxed{\begin{matrix} \ln 1 = 0 \\ \ln e = 0 \end{matrix}}$$

$$= (2+3) * 1 + (0)\ (\acute{g}(0)) = \underline{\underline{5}}$$

④ let $y = 2\cos t$, $t = \frac{\pi}{2}x^2$ find $\frac{dy}{dx}$ at $x=1$

Ⓐ $-2\pi$ Ⓑ $4\pi$ Ⓒ $-3\pi$ Ⓓ $6\pi$

⑤ let $f(x)$ be diff at $x=1$ and $f(x^3-7) = 6x^4$

find $\acute{f}(1)$

Ⓐ 16 Ⓑ $-4$ Ⓒ 2 Ⓓ 8

⑥ let $f(x) = 6\ln|x|$ find $\acute{f}(1)$

Ⓐ 12 Ⓑ 6 Ⓒ $-6$ Ⓓ 3

Nov. 8th. 2016

(7) let $f(x) = \ln x + e^{\sin(\pi x)}$ find $f'(1)$

$\boxed{(A)\ 1-\pi}$ (B) $1+\pi$ (C) $0$ (D) $\pi$

Sol(7)

$$f'(x) = \frac{1}{x} + e^{\sin(\pi x)} * \cos(\pi x) * \pi$$

$$f'(1) = \frac{1}{1} + e^{\sin(\pi)} * \cos(\pi) * \pi$$

$$= 1 + e^{0} * (-1) * \pi$$

$$= 1 - \pi$$

حفظ $\sin \pi = 0$, $\cos \pi = -1$

(8) let $y = \dfrac{x-1}{x+1}$ find $\dfrac{d^3y}{dx^3}$

(A) $12(x-1)^3(x+1)^{-3}$

(B) $12(x-1)(x+1)^{-4}$

$\boxed{(C)\ 12(x+1)^{-4}}$

(D) $12x^2(x+1)^{-3}$

Sol(8)

$\Rightarrow y = (x-1)(x+1)^{-1}$

$y' = (x-1)(-1)(x+1)^{-2} + (x+1)^{-1} * 1$

$= (1-x)(x+1)^{-2} + (x+1)^{-1}$

$y'' = (1-x)(-2)(x+1)^{-3} + (x+1)^{-2} * (-1) + (-1)(x+1)^{-2}$

$= -2(1-x)(x+1)^{-3} - (x+1)^{-2} - (x+1)^{-2}$

$y''' = 2\left[(x-1)(-3)(x+1)^{-4} + 2(x+1)^{-3} + 2(x+1)^{-3}\right] - 6(x-1)(x+1)^{-4} + 6(x+1)^{-3}$

$y''' = 6(x+1)^{-3}\left(-(x-1)(x+1)^{-1} + 1\right)$

$= 6(x+1)^{-3}\left(\dfrac{-(x-1)}{x+1} + \dfrac{x+1}{x+1}\right)$

$= 6(x+1)^{-3}\left(\dfrac{-x+1+x+1}{x+1}\right)$

$= 6(x+1)^{-3}\left(\dfrac{2}{x+1}\right)$

$= 12(x+1)^{-4}$

$$\begin{array}{r|l} & 1 \\ x+1 & x-1 \\ & \underline{-x \mp 1} \\ & -2 \end{array}$$

(8) Sol(8)

$$y = \frac{x-1}{x+1} = 1 + \frac{-2}{x+1}$$

$$y' = 0 + \frac{-(-2)(1)}{(x+1)^2} = \frac{2}{(x+1)^2}$$

$$y'' = \frac{-2(2)(x+1)}{(x+1)^4} = \frac{-4}{(x+1)^3}$$

$$y''' = \frac{-(-4)(3)(x+1)^2}{(x+1)^6} = \frac{12}{(x+1)^4}$$

$$= 12(x+1)^{-4}$$

(9) The derivative of $f(x) = \frac{1}{(\sqrt{x}+1)^2}$ is

(A) $\frac{-1}{\sqrt{x}(\sqrt{x}+1)}$

(B) $\frac{-1}{(\sqrt{x}+1)^3}$

(C) $\boxed{\frac{-1}{\sqrt{x}(\sqrt{x}+1)^3}}$

(D) $\frac{-1}{x(\sqrt{x}+1)^3}$

10. If $u = \sqrt{t}$ , $t = \ln x + x$ the $\frac{du}{dx}$ at $x = 1$

Ⓐ 3 Ⓑ 2 $\boxed{Ⓒ\ 1}$ Ⓓ $-2$

Sol (10) $\frac{du}{dx} = \frac{du}{dt} * \frac{dt}{dx}$

$= \left(\frac{1}{2\sqrt{t}}\right) * \left(\frac{1}{x} + 1\right)$

$\left.\frac{du}{dx}\right|_{x=1,\ t=1} = \frac{1}{2\sqrt{1}} * \frac{1}{1} + 1$

$= 1$

but $t = \ln x + x$

$= \ln 1 + 1$

$= 0 + 1$

$\boxed{t = 1}$

(11) let $y = 3^{\sin x} + \sec^2 x$ find $f'(0)$

$\boxed{Ⓐ\ \ln 3}$ Ⓑ $\frac{1}{\ln 3}$ Ⓒ 1 Ⓓ 0

(12) Find the slope of $y = 2^{\sin x}$ at $x = \pi$

(A) $\ln 2$ (B) $\frac{1}{\ln 2}$ (C) $1$ $\boxed{\text{(D) } -\ln 2}$

Sol (11) $y = f(x) = 3^{\sin x} + \sec^2 x$

$f'(x) = 3^{\sin x} * \ln 3 * \cos x + 2\sec^2 x * \sec x \tan x$

$f'(0) = 3^{\sin 0} * \ln 3 * \cos 0 + 2\sec^2 0 * \tan 0$

$= 3^0 * \ln 3 * 1 + 2(1)^2 * 0$

$= \ln 3$

$\sin 0 = 0$

$\cos 0 = 1$

$\sec 0 = \frac{1}{\cos 0} = 1$

$\tan 0 = \frac{\sin 0}{\cos 0} = \frac{0}{1} = 0$

Sol (12) slope $= f'(x)$

$= 2^{\sin x} * \ln 2 * \cos x$

$f'(\pi) = 2^{\sin \pi} * \ln 2 * \cos \pi$

$= -\ln 2$

سؤال
essay · Ex:- Find the equation of the tangent line to

① $f(x) = \frac{2x+1}{x+1} + \sin^3 x$ at $x = 0$

② $f(x) = \sin^3 x$ at $x = \frac{\pi}{4}$

Sol ②. Equation : $y - y_1 = m(x - x_1)$

$\Longrightarrow x_1 = \frac{\pi}{4}$ , $y_1 = ??$ , $m = f'(x_1)$

$\frac{\pi}{4} = 45^\circ$

$\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$\tan\frac{\pi}{4} = 1$

$$y_1 = f(x_1) = f\left(\frac{\pi}{4}\right)$$
$$= \sin^3\left(\frac{\pi}{4}\right)$$
$$= \left(\sin\frac{\pi}{4}\right)^3$$
$$= \left(\frac{1}{\sqrt{2}}\right)^3$$
$$\boxed{y = \frac{1}{2\sqrt{2}}}$$

$$\text{Slope} = f'(x)$$
$$= 3\sin^2 x * \cos x$$
$$f'\left(\frac{\pi}{4}\right) = 3\left(\sin\frac{\pi}{4}\right)^2 * \cos\frac{\pi}{4}$$
$$= 3\left(\frac{1}{\sqrt{2}}\right)^2 * \frac{1}{\sqrt{2}}$$
$$\boxed{m = \frac{3}{2\sqrt{2}}}$$

$$\boxed{y - \frac{1}{2\sqrt{2}} = \frac{3}{2\sqrt{2}}\left(x - \frac{\pi}{4}\right)}$$

$=(x^2+1)^{\frac{1}{2}}$

5 points

**Ex:** find $f'(x)$ for $f(x) = \ln\left(\dfrac{\cos x\sqrt{x^2+1}}{x^3}\right)$ using logarithm rules.

Sol

$$f(x) = \ln\cos x + \ln\sqrt{x^2+1} - \ln x^3$$
$$= \ln\cos x + \frac{1}{2}\ln(x^2+1) - 3\ln x$$

Recall

$\ln(xy) = \ln x + \ln y$

$\ln\dfrac{x}{y} = \ln x - \ln y.$

$\ln(x^n) = n\ln x$

$$f'(x) = \frac{-\sin x}{\cos x} + \frac{1}{2}\left(\frac{2x}{x^2+1}\right) - 3\left(\frac{1}{x}\right)$$

$$f'(x) = -\tan x + \frac{x}{x^2+1} - \frac{3}{x}.$$

Wed. Nov. 16th. 2016 ~~Oct. 31th. 2016~~

## * Domain of exp, log and Trig function.

**Remark** ① Domain of $f(x) = \left\{\begin{matrix} \sin g(x) \\ \cos g(x) \end{matrix}\right\} = \text{Domain } g(x)$

② Domain of $f(x) = b^{g(x)} = \text{Domain } g(x)$

③ Domain of $f(x) = \ln(g(x)) = \{x : g(x) > \underline{0}\}$

↳ لا يمكن ان يكون صفر لانه ln 0 غير موجود
(ln ← ارقام موجبة)

**Ex:-** Find the following domain.

① $f(x) = \sin\left(\frac{1}{x}\right)$ Domain $\mathbb{R} - \{0\}$

② $f(x) = 2^{\frac{x+1}{x-1}}$ Domain $\mathbb{R} - \{1\}$

اسئلة تمرين

③ $f(x) = \ln\left(\frac{1-x}{x}\right)$

④ $f(x) = \ln(3-x) - \ln(x+1)$

⑤ $f(x) = \frac{\sqrt{x}}{2^x - 4}$

**Sol (3).**

Domain of $f(x) = \{x : \frac{1-x}{x} > 0\}$

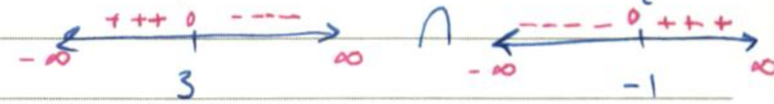
$$\frac{1-x}{x} > 0$$

Sign of $\frac{1-x}{x}$: $-\infty$ --- $\overset{u}{0}$ +++ $\overset{0}{1}$ --- $\infty$

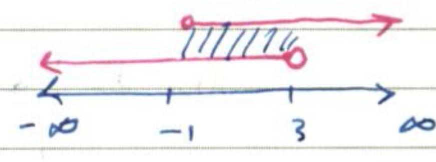
Domain of $f(x) \Longleftarrow$ is $(0, 1)$

Sol(4) Domain of $f(x)$ = Domain $\{\ln(3-x)\} \cap$ Domain $\{\ln(x+1)\}$

$3-x>0$




$(-\infty, 3) \cap (-1, \infty)$




Domain of $f(x)$ is $(-1, 3)$

Sol(5) Domain of $f(x)$ =
Domain $\{\sqrt{x}\} \cap$ Domain $\{2^x - 4\} - \{x : 2^x - 4 = 0\}$

$= x \geq 0 \cap \mathbb{R} - \{2\}$

$= \underbrace{[0, \infty) \cap \mathbb{R}} - \{2\}$

$= [0, \infty) - \{2\} \Rightarrow = [0,2) \cup (2, \infty)$

$2^x - 4 = 0$
$2^x = 4$
$2^x = 2^2$
$\boxed{x = 2}$

* Solving Equations involving Exp. and log Expressions :

* Exponential Rules and properties :

① $b^m * b^n = b^{m+n}$

② $\frac{b^m}{b^n} = b^{m-n}$

③ $(b^m)^n = b^{mn}$

④ $b^{-m} = \frac{1}{b^m}$

⑤ $\left(\frac{b}{a}\right)^{-m} = \left(\frac{a}{b}\right)^m$

⑥ $(ab)^m = a^m b^m$

⑦ $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

⑧ $b^{m/n} = \sqrt[n]{b^m}$

⑨ $b^0 = 1 \quad , b \neq 0$

## * Logarithm Rules and properties

① $\log_b(xy) = \log_b x + \log_b y$ if $x>0, y>0$

② $\log_b \frac{x}{y} = \log_b x - \log_b y$

③ $\log_b x^n = n \log_b x$

** ④ $b^{\log_b x} = x$

⑤ $\log_b b^x = x$

⑥ $\log_b b = 1$ , $\log_b 1 = 0$ , $\ln e = 1$ , $\ln 1 = 0$

⑦ $\log_b a = \frac{\log_c a}{\log_c b} = \frac{\ln a}{\ln b}$

⑧ $\log_c a * \log_b c = \log_b a$

** ⑨ $\log_{b^n} b^m = \frac{m}{n}$

** ⑩ (GOLDEN RULE) → $y = \log_b x \Longleftrightarrow x = b^y$

$\boxed{**}$ مهمين واكتر اشي بساعد.

(*) $e^{x^2} \neq e^{2x}$

$(e^x)^2 = e^{2x}$

Ex:- $3^{2^3} \neq (3^2)^3$

$3^8 \neq 9^3$

$(3^2)^3 = 3^6$

$9^3 = 3^6$ ✓

$\boxed{\text{Rule 7}}$ $\log_2 3 = \dfrac{\log_{10} 3}{\log_{10} 2} = \dfrac{\ln 3}{\ln 2}$

$\boxed{\text{Rule 8}}$ نفس (7)

$\boxed{\text{Rule 9}}$ $\log_9 \sqrt{27} = \log_{3^2} 3^{3/2} = \dfrac{3/2}{2} = \dfrac{3}{4}$

$\boxed{\text{Rule 10}}$ $8 = \log_2 x$

$\Longrightarrow x = 2^8$

$= 256$.

Ex1- Find the exact value. (4 points)

① $\log_2 512$ ③ $\log_{10} 100$

② $\log_{27} 243$ ④ $\log_2 0.125$

⑤ $\dfrac{\log_2 20 - \log_2 5}{\log_2 3 - \log_2 6}$

⑥ $(\sqrt{3})^{\log_3 9}$

⑦ $\log_{\frac{1}{3}} 27$

* ⑧ $3^{\log_{1/9} 16}$

* ⑨ $\sqrt{(\sqrt{2})}^{\log_2 81}$

اسئلة سنوات

Home work ← ⑩ $\log_2 3 + \log_2 6 - \log_2 (9/\sqrt{2})$

Sol(1) $\log_2 2^9 = 9 \log_2 2$ $\boxed{\text{OR}}$ $\log_{2^1} 2^9 = \frac{9}{1}$

$\boxed{= 9}$

| | |
|---|---|
| 2 | 512 |
| 2 | 256 |
| 2 | 128 |
| 2 | 64 |
| 2 | 32 |
| 2 | 16 |
| 2 | 8 |
| 2 | 4 |
| 2 | 2 |
| | 1 |

Sol(2) $\log_{3^3} 3^5 = \boxed{\frac{5}{3}}$

Sol(3) $\log_{10^1} 10^2 = \boxed{\frac{2}{1}}$

Sol(4) $\log_2 0.125 = \log_2 \frac{125}{1000}$

$2 = (\sqrt{2})^2$

$2 = \left(\frac{1}{2}\right)^{-1}$

$= \log_2 \frac{1}{8}$

$= \log_{2^1} 2^{-3} = \frac{-3}{1} = \boxed{-3}$

Sol(5) $\dfrac{\log_2 \frac{20}{5}}{\log_2 \frac{3}{6}} = \dfrac{\log_2 4}{\log_2 \frac{1}{2}} = \dfrac{\log_2 2^2}{\log_{2^1} 2^{-1}} = \dfrac{2}{-1}$

$\boxed{= -2}$

sol(6) $(\sqrt{3})^{\log_3 9} = (\sqrt{3})^{\log_{3^1} 3^2}$

$= (\sqrt{3})^2$

$= \boxed{3}$

OR $(\sqrt{3})^{\log_3 3^2} = (\sqrt{3})^{2\log_3 3}$

$= 3 \log_3 3$

$= 3$

Sol(7)

$\log_{3^{-1}} 3^3 = \frac{3}{-1} = \boxed{-3}$

Sol(8) $3^{\log_{9^{-1}} 4^2}$ = $\cancel{9^{\log_{9^{-1}} 4}}$

انتبه سؤال مهم

$= 3^{2\log_{9^{-1}} 4}$

$= 9^{\log_{9^{-1}} 4}$

$= \left(\frac{1}{9}\right)^{-1 \log_{\frac{1}{9}} 4} = \left(\frac{1}{9}\right)^{-\log_{\frac{1}{9}} 4} \Longrightarrow$

$$= \frac{1}{9}^{\log_{\frac{1}{9}} 4^{-1}}$$

$$= 4^{-1} = \frac{1}{4}$$

Sol (9) $(\sqrt{2})^{\log_2 81}$

$(\sqrt{2})^{\log_2 9^2}$

$$= \sqrt{2}^{2 \log_2 9}$$

$$= 2 \log_2 9 \qquad = \frac{9}{2} \times 2 \quad = 9$$

$\vdots$

$\boxed{\text{Mon. 21}^{\text{th}}\text{. Nov . 2016}}$

Ex Solve for $x$

① $\log_2 x + \log_2 (x+2) = 3$

② $\log_2 ( \log_3 (x+1)) = 2$

③ $e^{2x} = 9$

④ $2^{3x+4} = \frac{1}{16}x$

⑤ $3^x = 4$

⑥ $\log_3 (x+1) = 2$

⑦ $\ln x = 3$

$y = \log_b^x \Leftrightarrow x = b^y$

**Sol(1).** $\log_2 x\,(x+2) = 3$

$x(x+2) = 2^3 = 8$

$x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

$\Rightarrow x = -4$ $\boxed{\times}$ ✗ بالتعويض بالمسألة الأصلية

$\boxed{x = 2}$ ✓

**Sol(2).**

$\log_3 (x+1) = 2^2 = 4$

$x + 1 = 3^4$

$x + 1 = 81$

$\Rightarrow \boxed{x = 80}$

**Sol(3).** $(e^x)^2 = 9$

$(e^x)^2 - 9 = 0$

$(e^x - 3)(e^x + 3) = 0$

$e^x = 3$ $\qquad\qquad$ $e^x = -3$

$x = \log_e 3 = \ln 3$ $\boxed{\checkmark}$ $\qquad\qquad$ $x = \log_e -3$ $\boxed{\times}$

Sol(4) $2^{3x+4} = \frac{1}{16^x}$

$2^{3x+4} = \frac{1}{(2^4)^x} = 2^{-4x}$

$3x+4 = -4x$

$7x = -4$

$\boxed{x = \frac{-4}{7}}$

Sol(5) $x = \log_3 4$

Sol(6) $x+1 = 3^2$

$x+1 = 9$

$\boxed{x = 8}$

Sol(7) $\ln x = 3$

$\log_e x = 3$

$\boxed{x = e^3}$.

## * limits of trig function

Thm: $\lim_{x \to 0} \frac{\sin x}{x} = 1$

**Remarks:**

① $\lim_{x \to 0} \frac{\sin ax}{x} = a$

② $\lim_{x \to 0} \frac{\sin ax}{\sin bx} = \frac{a}{b}$

③ $\lim_{x \to 0} \frac{\tan ax}{x} = a$

④ $\lim_{x \to 0} \frac{\sin ax}{\tan bx} = \frac{a}{b}$

⑤ $\lim_{x \to 0} \frac{\tan ax}{\sin bx} = \frac{a}{b}$

⑥ $\lim_{x \to 0} \frac{\tan ax}{\tan bx} = \frac{a}{b}$

**Ex:** find

**Ans:-**

① $\lim_{x \to 0} \frac{\sin 3x}{x} = 3$

② $\lim_{x \to 0} \frac{\sin 4x}{\tan 5x} = \frac{4}{5}$

أسئلة امتحان ③ $\lim_{\theta \to 0} \frac{\tan^2 4\theta + \sin^2 5\theta}{\theta^2}$ →

④ $\lim_{x \to 0} \frac{\sin^2 4x}{x}$

** ⑤ $\lim_{x \to \pi} \frac{\sin 2x}{x} = \frac{\sin 2\pi}{\pi} = \frac{0}{\pi} = 0$

Sol (3) $\lim_{\theta \to 0} \frac{\tan^2 4\theta}{\theta^2} + \lim_{\theta \to 0} \frac{\sin 5\theta}{\theta^2}$

$$= \left(\lim_{\theta \to 0} \frac{\tan 4\theta}{\theta}\right)^2 + \left(\lim_{\theta \to 0} \frac{\sin 5\theta}{\theta}\right)^2$$

$$= 16 + 25 = \boxed{41}$$

Sol (4) $\lim_{x \to 0} \frac{\sin 4x}{x} * \lim_{x \to 0} \sin 4x$

$$= 4 * \sin 0$$
$$= 4 * 0$$
$$= 0$$

* Trig Identities متطابقات

① $\sin^2 x + \cos^2 x = 1$

② $\tan^2 x + 1 = \sec^2 x$

③ $1 + \cot^2 x = \csc^2 x$

④ $\sin 2x = 2 \sin x \cos x$

⑤ $\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x \rightarrow 1 - \cos 2x = 2\sin^2 x$

⑥ $\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x \rightarrow 1 + \cos 2x = 2\cos^2 x$

**Ex(2)** find

① $\lim\limits_{x\to 0} \dfrac{1-\cos x}{x^2}$

② $\lim\limits_{x\to 0} \dfrac{1-\cos 5x}{1-\cos 7x}$

اسئلة سنوات. ③ $\lim\limits_{\theta\to 0} \dfrac{1-\cos 2\theta}{\theta^2}$

④ $\lim\limits_{x\to 0} \dfrac{1+2\sin x+\sin^2 x}{\sqrt{4+\cos^3 x}}$

ممكن نحلها بطريقة ثانية مثل لوبتال

**Sol(1)** $\lim\limits_{x\to 0} \dfrac{1-\cos x}{x^2} * \dfrac{1+\cos x}{1+\cos x}$

$= \lim\limits_{x\to 0} \dfrac{1-\cos^2 x}{x^2(1+\cos x)}$

$= \lim\limits_{x\to 0} \dfrac{\sin^2 x}{x^2(1+\cos x)}$

$= \left(\lim\limits_{x\to 0} \dfrac{\sin x}{x}\right)^2 * \lim\limits_{x\to 0} \dfrac{1}{1+\cos x}$

$= (1)^2 * \dfrac{1}{2} = \boxed{\dfrac{1}{2}}$

طريقة ثانية

**Sol(2)**

$$\lim_{x \to 0} \frac{1 - \cos 5x}{1 - \cos 7x}$$

$$= \lim_{x \to 0} \frac{2 \sin^2\left(\frac{5x}{2}\right)}{2 \sin^2\left(\frac{7x}{2}\right)}$$

$$= \lim_{x \to 0} \frac{\sin^2\left(\frac{5x}{2}\right)}{\sin^2\left(\frac{7x}{2}\right)}$$

$$= \left(\frac{\frac{5}{2}}{\frac{7}{2}}\right)^2$$

$$= \frac{25}{49}$$

**Sol(3)**

$$\lim_{\theta \to 0} \frac{1 - \cos 2\theta}{\theta^2}$$

$$= \lim_{\theta \to 0} \frac{2 \sin^2 \theta}{\theta^2}$$

$$= 2$$

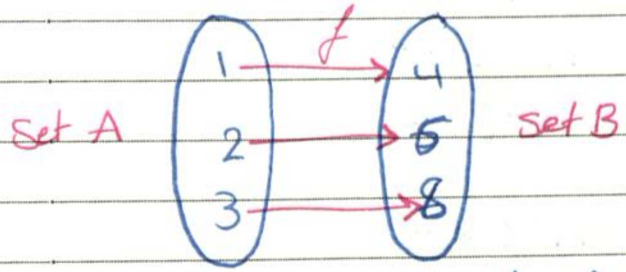
**Sol(4)**

$$= \frac{1 + 2 \sin 0 + \sin^2 0}{\sqrt{4 + \cos^2 0}}$$ تعويض مباشر

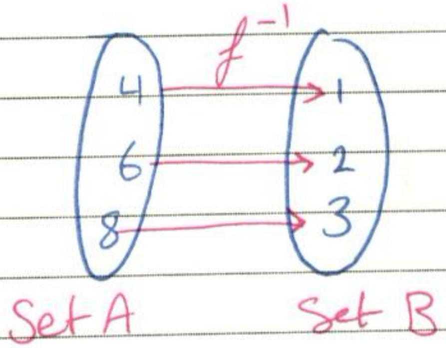
$$= \frac{1}{\sqrt{5}}$$

Wed. $23^{th}$. Nov. 2016

## * Inverse function ($f^{-1}$)




* Domain of $f = A$
* Range of $f = B$
* $f(1) = 4$ , $f(2) = 6$ , $f(3) = 8$
* $f$ is one-to-one




* Domain of $f^{-1} = B$
* Range of $f^{-1} = A$
* $f^{-1}(4) = 1$ , $f^{-1}(6) = 2$ , $f^{-1}(8) = 3$
* $f^{-1}$ is one-to-one.

## *Properties of Inverse function :-

① $f$ and $f^{-1}$ are one-to-one functions

② Domain of $f^{-1}$ = Range of $f$.

③ Range of $f^{-1}$ = Domain of $f$.

$f(f^{-1}(4))=4$
$f^{-1}(f(1))=1$

④ $f(f^{-1}(x)) = x$ ; $x \in$ Domain of $f^{-1}$

⑤ $f^{-1}(f(x)) = x$ ; $x \in$ Domain of $f$.

* Caution : $f^{-1}(x) \neq \frac{1}{f(x)}$

How we can find the inverse function?

Answer we have to follow the following Steps :-

step(1): Replace $f(x)$ with $y$.

Step(2): Interchange $x$ and $y$.

step(3): Solve for $y$.

Step(4): Replace $y$ with $f^{-1}(x)$.

Ex:- Find $f^{-1}(x)$ and Range of $f(x)$ for :-

H.W ① $f(x) = \dfrac{9x-1}{3x-6}$

② $f(x) = \log_2 (x+4)$

③ $f(x) = 2^x - 3$

H.W ④ $f(x) = \dfrac{e^{2x}+3}{3e^{2x}-1}$

Sol (2) Step(1): $y = \log_2 (x+4)$

Step(2): $x = \log_2 (y+4)$

Step(3): $y+4 = 2^x$

$\Longrightarrow y = 2^x - 4$

Step(4): $f(x)^{-1} = 2^x - 4.$

$\boxed{*}$ Range of $f(x)$ = Domain of $f^{-1}(x)$

$= \mathbb{R}$

**Sol (3).** **Step(1):** $y = 2^{x} - 3$

**Step(2):** $x = 2^{y} - 3$

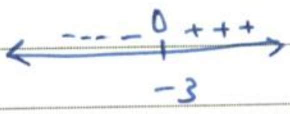
**Step(3):** $2^{y} = x + 3$

~~Step(4):~~ $y \cancel{\log_2 2} = \log_2 (x+3)$

$$y = \log_2 (x+3)$$

**Step(4):** $f^{-1}(x) = \log_2 (x+3)$

$\boxed{*}$ Range of $f(x)$ = Domain of $f^{-1}(x)$
$= (-3, \infty)$




استثنينا الـ 3- لأنه يصبح ما داخل الـ log = 0 نستثنيه.

**Ex** let $f(x) = x^3 + x + 1$ find

① $f^{-1}(11)$ ② $f^{-1}(1)$ ③ $f^{-1}(-1)$
④ $f^{-1}(31)$

$\boxed{*}$ لا نستطيع حلها على الخطوات السابقة لأنه عند الخطوة الثالثة لا يمكن فصل الـ $x$ عن الـ $y$.

Sol. By guessing بالتخمين

$$\boxed{f(a) = b \iff f^{-1}(b) = a}$$

① $f^{-1}(11) = 2$ معناه $\rightarrow f(2) = 11$

② $f^{-1}(1) = 0 \longrightarrow f(0) = 1$

③ $f^{-1}(-1) = -1 \longrightarrow f(-1) = -1$

④ $f^{-1}(31) = 3 \longrightarrow f(3) = 31$