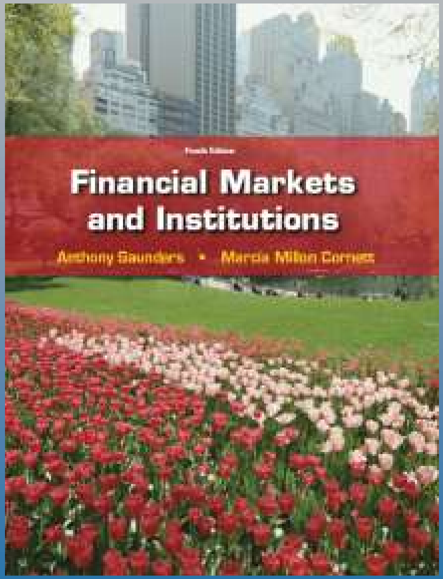
Chapter Three

Interest Rates and Security Valuation



Various Interest Rate Measures

- **Coupon rate**
  – periodic cash flow a bond issuer contractually promises to pay a bond holder
- **Required rate of return (*rrr*)**
  – rates used by individual market participants to calculate **fair present values (*PV*)**
- **Expected rate of return (*Err*)**
  – rates participants would earn by buying securities at **current market prices (*P*)**
- **Realized rate of return (*rr*)**
  – rates actually earned on investments

Required Rate of Return

- The **fair present value (*PV*)** of a security is determined using the **required rate of return (*rrr*)** as the discount rate

$$PV = \frac{\tilde{C}F_1}{(1+rrr)^1} + \frac{\tilde{C}F_2}{(1+rrr)^2} + \frac{\tilde{C}F_3}{(1+rrr)^3} + \ldots + \frac{\tilde{C}F_n}{(1+rrr)^n}$$

$\boldsymbol{CF_1}$ = cash flow in period $\boldsymbol{t}$ ($\boldsymbol{t}$ = 1, …, $\boldsymbol{n}$)

~ = indicates the projected cash flow is uncertain

$\boldsymbol{n}$ = number of periods in the investment horizon

Expected Rate of Return

- The **current market price (*P*)** of a security is determined using the **expected rate of return (*Err*)** as the discount rate

$$P = \frac{\tilde{C}F_1}{(1+Err)^1} + \frac{\tilde{C}F_2}{(1+Err)^2} + \frac{\tilde{C}F_3}{(1+Err)^3} + \ldots + \frac{\tilde{C}F_n}{(1+Err)^n}$$

CF_1 = cash flow in period ***t*** (***t*** = 1, …, ***n***)

~ = indicates the projected cash flow is uncertain

n = number of periods in the investment horizon

Realized Rate of Return

- The **realized rate of return (*rr*)** is the discount rate that just equates the **actual purchase price** ($\bar{P}$) to the present value of the realized cash flows (RCF_t) t ($t = 1, \ldots, n$)

$$\bar{P} = \frac{RCF_1}{(1+rr)^1} + \frac{RCF_2}{(1+rr)^2} + \frac{RCF_3}{(1+rr)^3} + \ldots + \frac{RCF_n}{(1+rr)^n}$$

Bond Valuation

- The **present value of a bond** (***V_b***) can be written as:

$$V_b = \frac{INT}{2}\sum_{t=1}^{2T}\left(\frac{1}{(1+i_d/2)}\right)^t + \frac{M}{(1+i_d/2)^{2T}}$$

$$= \frac{INT}{2}(PVIFA_{i_d/2,2T}) + M(PFIV_{i_d/2,2T})$$

M = the par value of the bond

INT = the annual interest (or coupon) payment

T = the number of years until the bond matures

i = the annual interest rate (often called **yield to maturity (ytm)**)

Bond Valuation

- A **premium bond** has a coupon rate (*INT*) greater then the required rate of return (*rrr*) and the fair present value of the bond (V_b) is greater than the face value (*M*)
- **Discount bond:** if $INT < rrr$, then $V_b < M$
- **Par bond:** if $INT = rrr$, then $V_b = M$

Equity Valuation

- The **present value of a stock (P_t)** assuming zero growth in dividends can be written as:

$$P_t = D / i_s$$

D = dividend paid at end of every year

P_t = the stock's price at the end of year t

i_s = the interest rate used to discount future cash flows

Equity Valuation

- The **present value of a stock** (***P**$_t$*) assuming constant growth in dividends can be written as:

$$P_t = \frac{D_0(1+g)^t}{i_s - g} = \frac{D_{t+1}}{i_s - g}$$

D_0 = current value of dividends
D_t = value of dividends at time t = 1, 2, …, ∞
g = the constant dividend growth rate

Equity Valuation

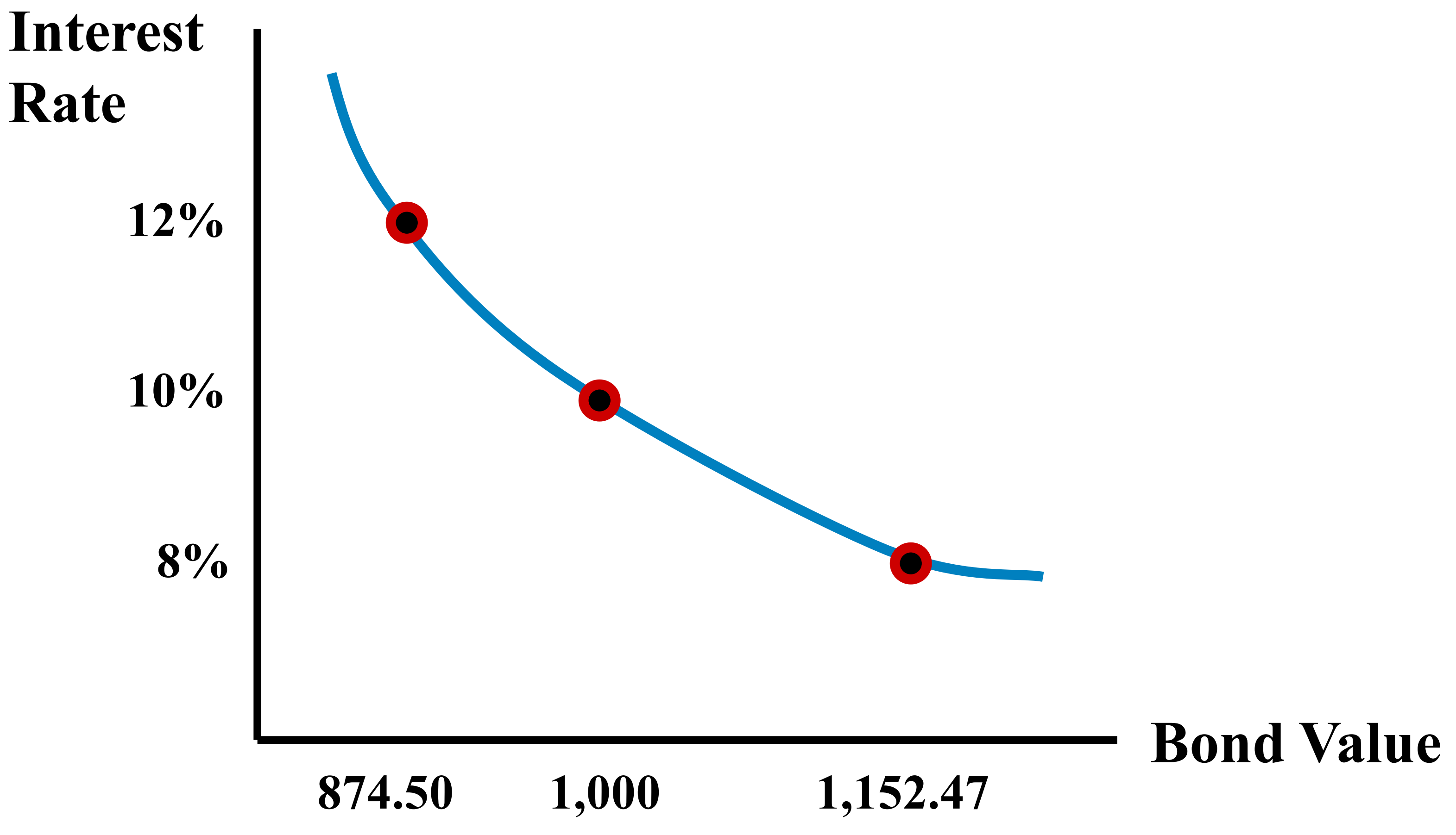
- The return on a stock with zero dividend growth, if purchased at price $\boldsymbol{P_0}$, can be written as:

$$i_s = D / P_0$$

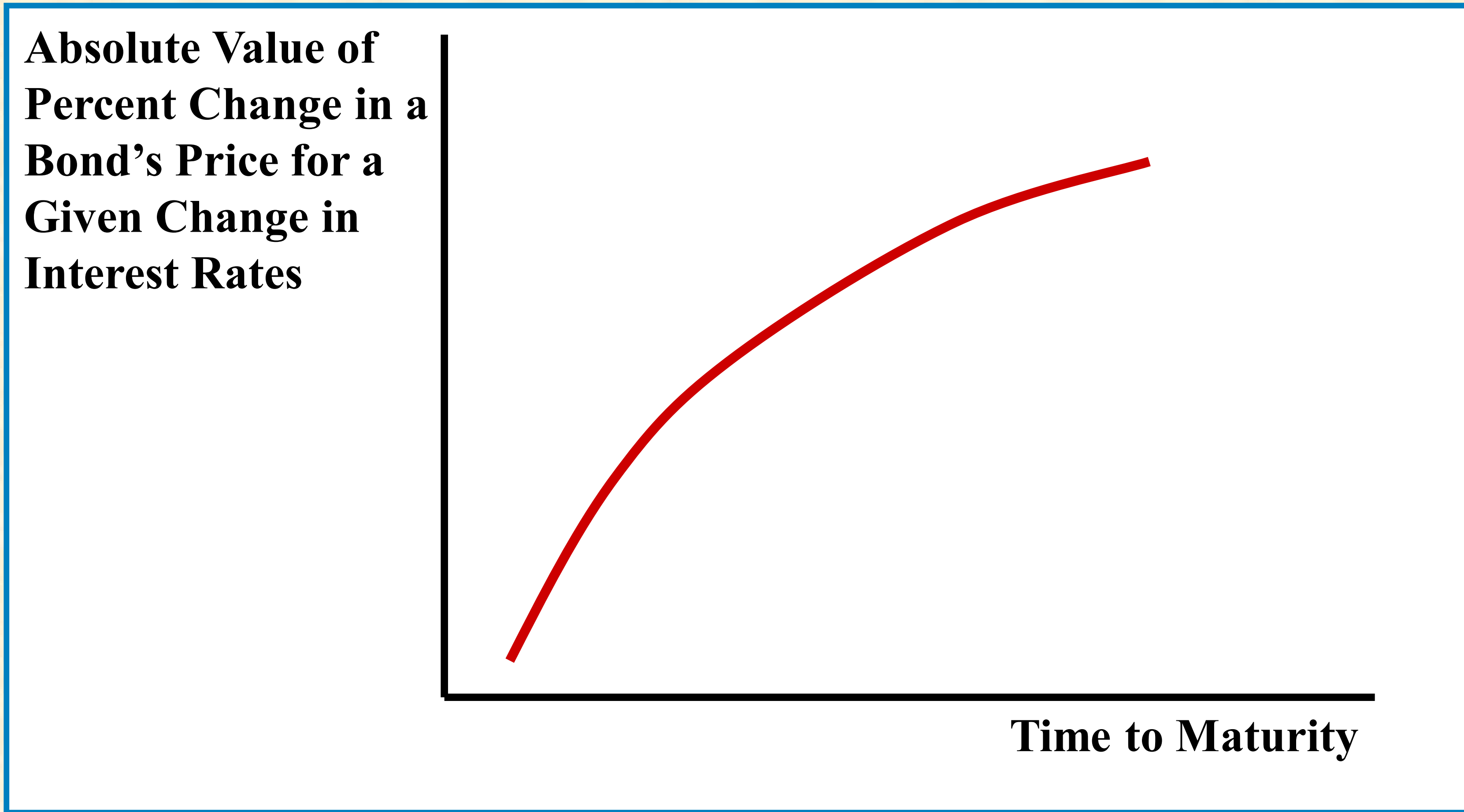
- The return on a stock with constant dividend growth, if purchased at price $\boldsymbol{P_0}$, can be written as:

$$i_s = \frac{D_0(1+g)}{P_0} + g = \frac{D_1}{P_0} + g$$

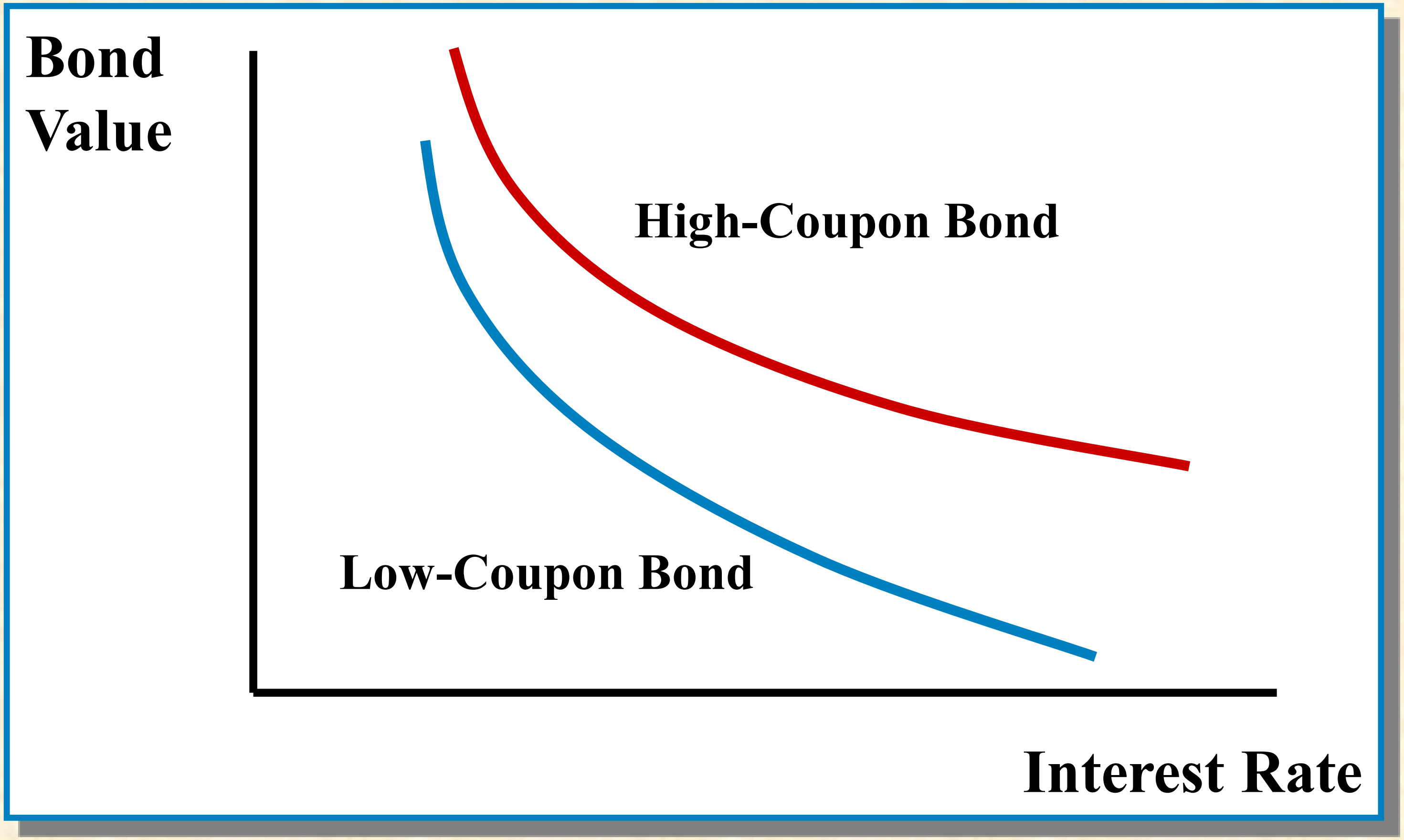
Relation between Interest Rates and Bond Values



Impact of Maturity on Interest Rate Sensitivity



Impact of Coupon Rates on Interest Rate Sensitivity



Duration

- **Duration** is the weighted-average time to maturity (measured in years) on a financial security
- **Duration** measures the sensitivity (or elasticity) of a fixed-income security's price to small interest rate changes

Duration

- **Duration (*D*)** for a fixed-income security that pays interest annually can be written as:

$$D = \frac{\sum_{t=1}^{T} \frac{CF_t \times t}{(1+R)^t}}{\sum_{t=1}^{T} \frac{CF_t}{(1+R)^t}} = \frac{\sum_{t=1}^{T} PV_t \times t}{\sum_{t=1}^{T} PV_t}$$

t = 1 to T, the period in which a cash flow is received

T = the number of years to maturity

CF_t = cash flow received at end of period t

R = yield to maturity or required rate of return

PV_t = present value of cash flow received at end of period t

Duration

- **Duration (*D*)** (measured in years) for a fixed-income security in general can be written as:

$$D = \frac{\displaystyle\sum_{t=1/m}^{T} \frac{CF_t \times t}{(1+R/m)^{mt}}}{\displaystyle\sum_{t=1/m}^{T} \frac{CF_t}{(1+R/m)^{mt}}}$$

m = the number of times per year interest is paid

Duration

- **Duration and coupon interest**
  - the higher the coupon payment, the lower the bond's duration
- **Duration and yield to maturity**
  - the higher the yield to maturity, the lower the bond's duration
- **Duration and maturity**
  - duration increases with maturity at a decreasing rate

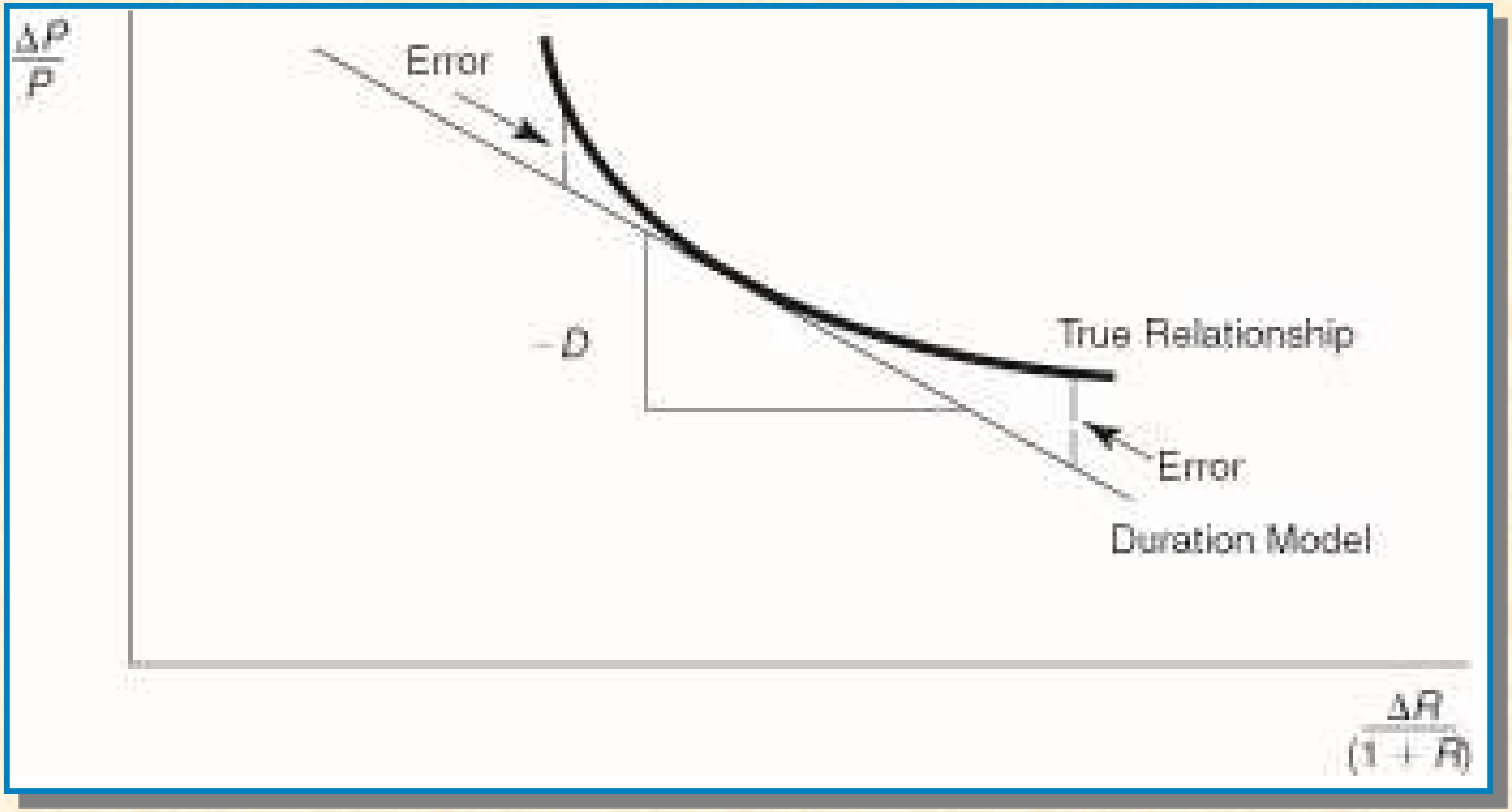
Duration and Modified Duration

- Given an interest rate change, the estimated percentage change in a (annual coupon paying) bond's price is found by rearranging the **duration** formula:

$$\frac{\Delta P}{P} = -D\left[\frac{\Delta R}{1+R}\right] = -MD \times \Delta R$$

MD = **modified duration** = $D/(1 + R)$

Figure 3-7



$\frac{\Delta P}{P}$
Error
−D
True Relationship
Error
Duration Model
$\frac{\Delta R}{(1+R)}$

Convexity

- **Convexity (*CX*)** measures the change in slope of the price-yield curve around interest rate level *R*
- **Convexity** incorporates the curvature of the price-yield curve into the estimated percentage price change of a bond given an interest rate change:

$$\frac{\Delta P}{P} = -D\left[\frac{\Delta R}{1+R}\right] + \frac{1}{2}CX(\Delta R)^2 = -MD \times \Delta R + \frac{1}{2}CX(\Delta R)^2$$