EE Dept., JUST, Spring 2015, 3/5/2015, Electromagnetics I, EE 207, Second Exam

Start each question in a new page. Write (a), (b), (c), ... Show your work. Put the final answer in a box. You may directly use any expression from the sheet. Good luck!

1. **(9 points)** A material (having a dielectric strength of 6 MV/m and ϵ_r=9) is placed inside a coaxial capacitor which is connected to a battery as shown in the figure below. The dimensions of the capacitor are a=1 mm, b=2 mm and L=10 cm. If the voltage applied is V=10 V, then the current is found to be I=2 A.

   (a) (2 points) Calculate the conductivity of the material filling the capacitor.

   (b) (3 points) Calculate the surface charge densities on the inner and outer conductors. (V=10 V)

   (c) (2 points) Calculate the maximum surface charge density on the outer conductor before breakdown.

   (d) (2 points) Calculate the breakdown voltage.

2. **(9 points)** A solid perfectly conducting sphere of radius 1 meter has an unknown uniform surface (free) charge density ρ_s. The sphere is surrounded by a perfect dielectric that extends from r=1 to r=5 meters.

   At r=2 meters, $\bar{D}= \hat{r}\,\epsilon_o/2$ (C/m^2). Moreover, at r=4 meters, $\bar{P}= \hat{r}\,\epsilon_o/16$ (C/m^2).

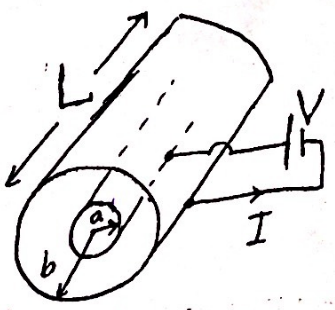
   (a) (2 points) Find ρ_s.

   (b) (2 points) Find ϵ_r of the dielectric region.

   (c) (3 points) Find the potential of the conducting sphere.

   (d) (2 points) Find the total electric energy W_e.

3. **(7 points)** A non-uniform volume charge density $\rho_v = \epsilon_0\, z^2$ (C/m^3) exists in the region defined as: $-1 < z < 1$. Solving the appropriate differential equations (i.e., Poisson's or Laplace's equations) and applying the appropriate boundary conditions, find expressions for the electric field intensity $\bar{E}$ in the regions: $z > 1$ (call it $\bar{E}_1$), $-1 < z < 1$ (call it $\bar{E}_2$), and $z < -1$ (call it $\bar{E}_3$). Assume ϵ=ϵ_0 everywhere.



$$\bar{\nabla} V = \frac{\partial V}{\partial x}\,\hat{x} + \frac{\partial V}{\partial y}\,\hat{y} + \frac{\partial V}{\partial z}\,\hat{z} \qquad \bar{\nabla} V = \frac{\partial V}{\partial \rho}\,\hat{\rho} + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\,\hat{\phi} + \frac{\partial V}{\partial z}\,\hat{z}$$

$$\bar{\nabla} V = \frac{\partial V}{\partial r}\,\hat{r} + \frac{1}{r}\frac{\partial V}{\partial \theta}\,\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\,\hat{\phi}$$

$$\bar{\nabla}\cdot\mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \qquad \bar{\nabla}\cdot\mathbf{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\bar{\nabla}\cdot\mathbf{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} \qquad \nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$

$$\bar{\nabla}\times\mathbf{A} = \hat{x}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{y}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{z}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\bar{\nabla}\times\mathbf{A} = \hat{\rho}\left(\frac{1}{\rho}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right) + \hat{\phi}\left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right) + \hat{z}\,\frac{1}{\rho}\left[\frac{\partial(\rho A_\phi)}{\partial \rho} - \frac{\partial A_\rho}{\partial \phi}\right]$$

$$\bar{\nabla}\times\mathbf{A} = \frac{\hat{r}}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(A_\phi\sin\theta) - \frac{\partial A_\theta}{\partial \phi}\right] + \frac{\hat{\theta}}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{\partial}{\partial r}(rA_\phi)\right] + \frac{\hat{\phi}}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial \theta}\right]$$

Here are some other expressions:

$\oint_s \bar{D}\cdot d\bar{s} = Q_{enc}$ $\quad \bar{\nabla}\cdot\bar{D} = \rho_v$ $\quad \bar{E} = \int \frac{dq}{4\pi\epsilon R^2}\,\hat{a}_R$ $\quad V = \int \frac{dq}{4\pi\epsilon R}$

$\bar{\nabla}\times\bar{E} = 0$ $\quad \bar{E} = -\bar{\nabla}V$ $\quad V_b - V_a = -\int_a^b \bar{E}\cdot d\bar{\ell}$ $\quad \bar{D} = \epsilon\bar{E}$

Electric dipole: $V = \frac{\bar{p}\cdot\hat{r}}{4\pi\epsilon r^2}$ $\quad \bar{E} = \frac{p}{4\pi\epsilon r^3}\,(\hat{r}2\cos\theta + \hat{\theta}\sin\theta)$

$W_e = \frac{1}{2}\sum Q_i V_i$ $\quad W_e = \frac{1}{2}\int \rho_v V\,dv$ $\quad W_e = \frac{1}{2}\epsilon\int |\bar{E}|^2\,dv$ $\quad W_e = \frac{1}{2}\,C\,V^2$

$\bar{J} = \sigma\bar{E}$ $\quad I = \int \bar{J}\cdot d\bar{s}$ $\quad \rho_{ps} = \bar{P}\cdot\hat{a}_n$ $\quad \rho_{pv} = -\bar{\nabla}\cdot\bar{P}$ $\quad \bar{D} = \epsilon_0\bar{E} + \bar{P}$ $\quad \bar{\nabla}\cdot\bar{J} = -\frac{\partial\rho_v}{\partial t}$

$\hat{a}_n\times(\bar{E}_1 - \bar{E}_2) = zero$ $\quad \hat{a}_{n1}\cdot(\bar{D}_2 - \bar{D}_1) = \rho_s$

$\bar{\nabla}\cdot(\epsilon\,\bar{\nabla}V) = -\rho_v$ $\quad \bar{\nabla}\cdot(\sigma\,\bar{\nabla}V) = 0$ $\quad$ Dissipated power $= \int \sigma E^2\,dv$

$RC = \epsilon/\sigma$ $\quad$ Coaxial: $C = \frac{2\pi\epsilon}{\ln(b/a)}$ (F/m) $\quad$ Spherical capacitor: $C = \frac{4\pi\epsilon}{1/a - 1/b}$ (F)

$\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2(x^2+a^2)^{1/2}}$ $\quad \int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$

$\int \frac{dx}{(x^2+a^2)^{1/2}} = \ln(x + \sqrt{x^2+a^2})$ $\quad \int \frac{x\,dx}{(x^2+a^2)^{3/2}} = \frac{-1}{(x^2+a^2)^{1/2}}$