Jordan University of Science & Technology - Electrical Engineering Department

EE301 - Applied Probability Theory - Midterm Exam # 1. Su. 97.07.13

Answer All Questions

1. **Conditional Probability:** A missile can be accidentally launched if two relays A and B both have failed. The probabilities of A and B failing are known to be 0.01 and 0.03 respectively. It is also known that B is more likely to fail (probability 0.06) if A has failed.
   (a) What is the probability of an accidental missile launch?
   (b) What is the probability that A will fail if B has failed?
   (c) Are the events "A fails" and "B fails" statistically independent?

2. **Mean and Variance:** Find the mean and variance of the **Rayleigh** probability density functions (show all work)

$$f_X(x) = \begin{cases} \frac{2}{b}(x-a)e^{-(x-a)^2/b}, & x \geq a \\ 0, & x < a \end{cases}$$

   Hint: $\int_{-\infty}^{\infty} e^{-a^2x^2+bx}dx = \frac{\sqrt{\pi}}{a}e^{b^2/(4a^2)}, a > 0$ and $\int_0^{\infty} x^2e^{-x^2}dx = \frac{\sqrt{\pi}}{4}$

3. **Independent Events, Sets, Characteristic Function, ... :**
   (a) Let A be an arbitrary event. Show that $P(\bar{A}) = 1 - P(A)$.
   (b) **Indicator Variable**: Let I be defined as

$$I = \begin{cases} 1, & \text{if } A \text{ occurs} \\ 0, & \text{if } \bar{A} \text{ occurs} \end{cases}$$

   Find the expected value of I.
   (c) Show that $C \subset A$ if $C \subset B$ and $B \subset A$.
   (d) **Geometric Random variable:** The r.v. X having the probability

$$P(X = k) = \alpha(1-p)^{k-1}p, n = 1, 2, \cdots$$

   What should be the value of α
   (e) Let X be such that

$$P(X = 1) = p = 1 - P(X = -1)$$

   Find $c \neq 1$ such that $E[c^X] = 1$

4. In a school where 4% of the children write with their left hands, what is the probability that there are no left-handed children in a class of 25?

Grade distribution: 3*2+5+5*2+4=25

Dr. Abdel-Rahman Jaradat **Time: 50 minutes**