Jordan University of Science & Technology - EE Dept.

EE301 - Applied Probability Theory - Exam # 2. Mon. 97.08.04

<u>**Answer All Questions**</u>

1. **Computer generation of different distributions:** Show that given a uniformly distributed random number X, one can get a Rayleigh random number via the transformation $Y = T(x) = \sqrt{-b\ln(1-x)}$, for $0 < x < 1$ and setting $a = 0$. Hint: recall that pdf for Rayleigh
$$f_X(x) = \frac{2}{b}(x-a)e^{\frac{-(x-a)^2}{b}}u(x-a)$$

2. **Chebyshev's Inequality:** For any real random variable X with mean $\bar{X}$ and variance σ_X^2, then
$$P\{|X-\bar{X}| \geq \lambda\sigma_X\} \leq \frac{1}{\lambda^2}$$
where $\lambda > 0$ is a real constant.

3. For the independent random variables X and Y show that
$$P\{Y \leq X\} = \int_{-\infty}^{\infty} F_Y(x) f_X(x) dx$$

4. **Central Limit Theorem:** Three statistically independent random variables X_1, X_2 and X_3 are defined by $\bar{X}_1 = -1, \sigma_{X_1}^2 = 2.0, \bar{X}_2 = 0.6, \sigma_{X_2}^2 = 1.5, \bar{X}_2 = 1.8, \sigma_{X_3}^2 = 0.8$ Write the equation describing the gaussian approximation for the density function of the sum $X = X_1 + X_2 + X_3$.

5. Gaussian random variables X_1 and X_2, for which $\bar{X}_1 = 2, \sigma_{X_1}^2 = 9, \bar{X}_2 = -1, \sigma_{X_2}^2 = 4$, and $C_{X_1X_2} = -3$, are transformed to new random variables Y_1 and Y_2 according to $Y_1 = -X_1 + X_2; Y_2 = -2X_1 - 3X_2$. Find $C_{Y_1Y_2}$.

6. $X(t)$ and $Y(t)$ are statistically independent, zero-mean random processes with autocorrelation functions $R_{XX}(\tau) = e^{-|\tau|}$ and $R_{YY}(\tau) = \cos(2\pi\tau)$ respectively. Find the cross correlation function of $W_1(t)$ and $W_2(t)$ where $W_1(t) = X(t) + Y(t)$ and $W_2(t) = X(t) - Y(t)$.

<u>**Grade distribution: 5+4+4+4+4+4=25**</u>

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