The exam has 42 questions, for a total of 42 points.

**Answer All Questions**

1. The largest absolute value of the joint characteristic function for $N$ random variables
   (a) is bound by the largest value of the joint pdf
   (b) =1
   (c) is bound by the largest values of its marginal pdfs
   (d) =0
   (e) = the value of the pdf at the origin

2. Three uncorrelated r.v.s $X_1, X_2$ and $X_3$ have means $\bar{X}_1 = 1, \bar{X}_2 = -3$, and $\bar{X}_3 = 1.5$ and second moments $E[X_1^2] = 2.5, E[X_2^2] = 11$, and $E[X_3^2] = 3.5$. Let $Y = X_1 - 2X_2 + 3X_3$ be a new r.v., its mean=
   (a) -0.5
   (b) 0
   (c) 0.5
   (d) 2.0
   (e) 11.5

3. In the above question, the variance of $Y$=
   (a) 4.75
   (b) 9.25
   (c) 9.5
   (d) 20.75
   (e) 41.5

4. Three statistically independent random variables $X$, $Y$, and $Z$ having means and variances as follows: $\bar{X} = -1, \sigma_X^2 = 2.0, \bar{Y} = 0.6, \sigma_Y^2 = 1.5, \bar{Z} = 1.8, \sigma_Z^2 = 0.8$. Using central limit theorem, the r.v. $W = X + Y + Z$ have a gaussian statistics of mean and variance equal to
   (a) 1.4 and 8.6
   (b) 2.4 and 8.6
   (c) $\frac{1.4}{3}$ and $\frac{4.3}{3}$
   (d) 1.4 and 4.3
   (e) 2.4 and 4.3

5. Two random variables $X$ and $Y$ have a joint pdf

$$f_{X,Y}(x,y) = \begin{cases} \frac{5}{16}x^2y, & 0 < y < x < 2 \\ 0, & elsewhere \end{cases}$$

   $X$ and $Y$
   (a) are statistically independent
   (b) are not statistically independent
   (c) have same mean
   (d) have same variance
   (e) have the same second moment

6. Given the function

$$f_{X,Y}(x,y) = \begin{cases} (x^2+y^2)/(8\pi), & x^2+y^2 < 4 \\ 0, & elsewhere \end{cases}$$

   Then $P\{2 < X^2 + Y^2 \leq 3.2\} =$
   (a) 0.12
   (b) 0.39
   (c) 0.5
   (d) 0.61
   (e) 0.88

7. The function

$$G_{X,Y}(x,y) = \begin{cases} 0, & x < y \\ 1, & x \geq y \end{cases}$$

   is a valid distribution function.
   (a) True
   (b) False

8. If $X$ and $Y$ are independent, then for all $y$ we have $E[X|Y = y] =$
   (a) $E[X] - E[Y]$
   (b) $E[Y|X = x]$
   (c) $\bar{Y}$
   (d) $E[X]$
   (e) $f_X(X|Y = y)$

9. If $Y = a + bX$, then $\rho(X,Y) = 1$ if
   (a) $b > 1$
   (b) $b > 0$
   (c) $a = b$
   (d) $b < 0$
   (e) $a = -b$

10. If $X$ and $Y$ are identically distributed, not necessarily independent, then $Cov(X+Y, X-Y) =$
    (a) $Cov(X, X-Y)$
    (b) $\sigma_X^2 + \sigma_Y^2$
    (c) $Cov(X+Y, X)$
    (d) 0
    (e) $Cov(Y, X-Y)$

11. The power density spectrum of a r.p. $X(t)$ is given by $S_{XX}(\omega) = Ae^{-\omega^2/(2\omega_o^2)}$. Its autocorrelation function is given by
    (a) $R_{XX}(\tau) = \frac{A\omega_o}{\sqrt{2\pi}} e^{-(\omega_o\tau^2)/2}$
    (b) $R_{XX}(\tau) = \frac{A\omega_o}{\sqrt{2\pi}} e^{-(\omega_o\tau)/2}$
    (c) $R_{XX}(t, t+\tau) = \frac{A\omega_o}{\sqrt{2\pi}} e^{-(\omega_o(t-\tau)^2)/2}$
    (d) $R_{XX}(\tau) = \frac{A\omega_o}{\sqrt{2\pi}} e^{-(\omega_o^2\tau^2)/2}$
    (e) $R_{XX}(\tau) = \frac{A\omega_o}{\sqrt{2\pi}} (1 - (\omega_o\tau^2)^2)$

12. The power density spectrum of a r.p. $X(t)$ is given by $S_{XX}(\omega) = A$ for $-\omega_o < \omega < \omega_o$ and zero elsewhere. Its autocorrelation function is given by $R_{XX}(\tau) = \frac{A}{\pi\tau} sin(\omega_o\tau)$
    (a) true
    (b) false

13. If $X(t)$ and $Y(t)$ are two independent random processes with power density spectra $S_{XX}(\omega)$ and $S_{YY}(\omega)$, the power density spectrum of their linear combination $Z(t) = aX(t) + bY(t)$ is
    (a) $S_{ZZ}(\omega) = S_{XX}(\omega) + S_{YY}(\omega)$
    (b) $S_{ZZ}(\omega) = 2ab(S_{XX}(\omega) + S_{YY}(\omega))$
    (c) $S_{ZZ}(\omega) = a^2S_{XX}(\omega) - b^2S_{YY}(\omega)$
    (d) $S_{ZZ}(\omega) = aS_{XX}(\omega) + bS_{YY}(\omega)$
    (e) $S_{ZZ}(\omega) = a^2S_{XX}(\omega) + b^2S_{YY}(\omega)$

14. The output $Y(t)$ of a linear system is $k$ times the input $X(t)$. Then
    (a) $R_{YY} = k^2 R_{XX}$
    (b) $R_{YY} = kR_{XX}$
    (c) $R_{YY} = R_{XX}$
    (d) $R_{YY} = R_{XX}/k$
    (e) $R_{YY} = R_{XX}/k^2$

15. The characteristic function for a r.v. $X$ having exponential pdf is given by $\Phi_X(\omega) = \frac{1}{1-ja\omega}$, then its variance is
    (a) $-a^2$
    (b) $-a$
    (c) $ja^2$
    (d) $a$
    (e) $a^2$

16. When a fair coin is tossed, the possible outcomes, heads and tails, are represented by a random variable $X$ which has the value 1 or -1, respectively. The characteristic function equals
    (a) $(1 + e^{j\omega})/2$
    (b) $j\sin(\omega)$
    (c) $-j\sin(\omega)$
    (d) $-\cos(\omega)$
    (e) $\cos(\omega)$

17. Find $k$ for the joint pdf
    $$f_{X,Y}(x,y) = \begin{cases} ke^{-2x-3y}, & x, y \geq 0 \\ 0 & else \end{cases}$$
    (a) 6
    (b) 1
    (c) 1/2
    (d) 1/3
    (e) 1/6

18. Find $\mu_{11}$ for the joint pdf
    $$f_{X,Y}(x,y) = \begin{cases} ke^{-(x+y)}, & x, y \geq 0 \\ 0 & else \end{cases}$$
    (a) -1
    (b) 0
    (c) 1/2
    (d) 1
    (e) $\infty$

19. A random variable x is uniformly distributed between -1 and +1. Determine the normalized correlation coefficient for $x$ and $y$ if $y = x^2$.
    (a) -1
    (b) -8/12
    (c) 0
    (d) 1
    (e) 4/12

20. A random variable $X$ has a probability density function $f_X(x) = \begin{cases} e^{-x}, & x \geq 0 \\ 0 & else \end{cases}$ Determine the probability density function for the random variable $Y = X^3$.
    (a) $e^{-y}/3y^2$
    (b) $e^{-y^{1/3}}/3y^{2/3}$
    (c) $2e^{-y}/y$
    (d) $e^{-y^3}/3y^2$
    (e) $e^{y^{1/3}}/3y^{-2/3}$

21. The magnitude of the characteristic function of a random variable $X$ is
    (a) $\geq 1.0$
    (b) $\geq \bar{X}$
    (c) $\leq 1.0$
    (d) $\leq \sigma_X$
    (e) $\leq \sigma_X^2$

22. Chebyshev's inequality states that any random variable $X$ has a finite second moment, then for any constant $C > 0$ we have
    (a) $P(|X| \geq c) \leq \frac{E(X^2)}{c^2}$
    (b) $P(|X| \leq c) \leq \frac{E(X^2)}{c^2}$
    (c) $P(|X| \geq c) \leq c^2 E(X^2)$
    (d) $P(|X| \geq c) \leq \frac{E(X)}{c^2}$
    (e) $P(|X| \leq c) \leq c^2 E(X^2)$

23. An analogue of Chebyshev's inequality where the absolute first moment is used in place of the second moment can be written as $P(|X - m|) > c)$
    (a) $\geq E(|X - m|)$
    (b) $\leq E(|X - m|)$
    (c) $\leq E(|X|)$
    (d) $\leq E(X^2)$
    (e) $\geq E(|X|)$

24. For two independent random variable $X$ and $Y$, with known individual pdf's, then any new random variable formed directly from their sum; i.e., $S = X + Y$; has a pdf equal to
    (a) $f_S(s) = f_X(x) f_Y(y)$
    (b) $f_S(s) = \int_{-\infty}^{\infty} f_Y(y) f_X(s - y) dy$
    (c) $f_S(s) = f_X(x) + f_Y(y)$
    (d) $f_S(s) = \frac{f_X(x)}{f_Y(y)}$
    (e) $f_S(s) = \int_{-\infty}^{\infty} f_X(x) f_Y(s - y) dx$

25. A geometric random variable $X$ has the probability $P(X = k) = pq^k, k = 0, 1, 2, \cdots, p + q = 1$, its expected value is
    (a) $\frac{1}{p}$
    (b) $pq$
    (c) $\frac{p}{q}$
    (d) $kpq$
    (e) $\frac{q}{p}$

26. A r.v. $X$ drawn from a Pascal distribution (or negative binomial),

$$P[X = k] = \binom{n + k - 1}{k} p^n q^k$$

, $k = 0, 1, \cdots, p + q = 1$. Its mean value=
    (a) $nq/p$
    (b) $q/p$
    (c) $p/q$
    (d) $np/q$
    (e) $kp/q$

27. For any r.v. $X$ with characteristic function $\Phi(\omega) = E[e^{j\omega X}]$, then for any collection of $n$ complex numbers $a_i$ we have $E[|\sum_{i=1}^{n} a_i e^{j\omega_i X}|^2] \geq 0$, one can write $\sum_{i=1}^{n} \sum_{k=1}^{n} \Phi(\omega_i - \omega_k) a_i a_k^* \geq 0$
    (a) true
    (b) false

28.

$$f_{X,Y}(x,y) = \begin{cases} \frac{15}{2}x(2-x-y) & 0 < x, y < 1 \\ 0 & \text{otherwise} \end{cases}$$

then the conditional pdf of $X$, given that $Y = y$ equals to

(a) $\frac{6(2-x-y)}{4-3y}$

(b) $\frac{6x(2-x-y)}{4-3y}$

(c) $\frac{x(2-x-y)}{6(4-3y)}$

(d) $\frac{6x^2(2-x-y)}{3y}$

(e) $\frac{x(2-x-y)}{4}$

29. The indicator variable $I$ for an event $A$ is defined as

$$I = \begin{cases} 1, & \text{if } A \text{ occurs} \\ 0, & \text{if } \bar{A} \text{ occurs} \end{cases}$$

We can write

(a) $P(I) = E[A]$

(b) $E[I] = P(A)$

(c) $Var[I] = E[A]$

(d) $Var[A] = E[I]$

(e) $P(\bar{A}) = E[I]$

30. In a group of resistors where 4% are of metal type resistors, what is the probability that there are no metal resistors in a random selection of 25 resistors?

(a) $1.0 - (0.04)^{25}$

(b) $1 - (0.96)^{25}$

(c) $(0.04)^{25}$

(d) 0.96

(e) $(1 - 0.04)^{25}$

31. Given that

$$f_{X|Y}(X|Y) = \begin{cases} ye^{-xy} & 0 < x < \infty, 0 < y < 2 \\ 0 & \text{otherwise} \end{cases}$$

then $E[e^{x/3}|Y = 1]$ is

(a) 0

(b) 1/3

(c) 3/2

(d) 2

(e) 2/3

32. Let $X$ be a r.v. and $c$ be a real constant number, let $Y = X + c$, then $\sigma_Y^2 =$

(a) $c^2\sigma_X^2$

(b) $\sigma_X^2 + c$

(c) $\sigma_X^2 + c^2$

(d) $\sigma_X^2$

(e) $\sigma_X^2 - c^2$

33. If two points $X$ and $Y$ are picked at random from the interval [0,1], the probability that the distance between them $P(|X - Y| < \frac{1}{2}) =$

(a) 1/8

(b) 1/4

(c) 3/8

(d) 1/2

(e) 3/4

34. Let $X_1, X_2, \cdots, X_n$ be independent random variables with cdfs $F_1, F_2, \cdots, F_n$. Let $M = maximum(X_1, X_2, \cdots, X_n)$ and $m = minimum(X_1, X_2, \cdots, X_n)$. The distribution for $M$ can be written as $F_{max}(x) = P(M \leq x) = P(X_1 \leq x; X_2 \leq x; \cdots; X_n \leq x) = P(X_1 \leq x)P(X_2 \leq x) \cdots P(X_n \leq x) = F_1(x)F_2(x) \cdots F_n(x)$. Following the arguments mentioned one can write the cdf for $m$, the minimum, as

(a) $F_{min}(x) = \Pi_{i=1}^n (1 - F_i(x))$

(b) $F_{min}(x) = 1 - \Pi_{i=1}^n F_i(x)$

(c) $F_{min}(x) = \Pi_{i=1}^n F_i(x)$

(d) $F_{min}(x) = 1 - \sum_{i=1}^n (1 - F_i(x))$

(e) $F_{min}(x) = 1 - \Pi_{i=1}^n (1 - F_i(x))$

35. Ergodic process $X(t)$ is a stochastic process that

(a) is second-order stationary

(b) has statistical averages = time averages

(c) is strictly stationary

(d) is first-order stationary

(e) has $\bar{X}$ =constant, and $R_{XX}(\tau)$=constant.

36. If $X(t)$ is WSS, and $Z(t) = aX(t)$ then
    (a) $C_{ZZ}(\tau) = |a|^2 C_{XX}(\tau)$
    (b) $R_{XZ}(\tau) = |a|^2 R_{XX}(\tau) R_{ZZ}(\tau)$
    (c) $R_{ZX}(\tau) = |a|^2 R_{XX}(\tau) R_{ZZ}(\tau)$
    (d) $R_{ZZ}(\tau) = |a|^2 R_{XX}(\tau)$
    (e) $C_{XZ}(\tau) = |a|^2 C_{ZX}(\tau)$

37. A r.v. $X$ drawn from a binomial distribution, i.e. $P(X = k) = \begin{pmatrix} N \\ k \end{pmatrix} p^k (1-p)^{N-k}$ then its variance =
    (a) $Np(1-p)$
    (b) $Np$
    (c) $N(1-p)$
    (d) $(N-1)p(1-p)$
    (e) $N^2 p(1-p)$

38. R.V. $X$ has

$$f_X(x) = \begin{cases} p & x = 1 \\ 1-p & x = -1 \end{cases}$$

    Its characteristic function =
    (a) $p + (1-p)e^{j\omega}$
    (b) $e^{-j\omega} + 2jp\sin(\omega)$
    (c) $(1-p)e^{j\omega} + pe^{-j\omega}$
    (d) $(1-p)e^{-j\omega}$
    (e) $pe^{j\omega}$

39. For the Poisson distribution, $P_X(X = k) = \frac{e^{-\lambda}\lambda^k}{k!}$ its mean $= \lambda$ and its variance =
    (a) $\lambda^2$
    (b) $\frac{1}{\lambda}$
    (c) $k\lambda$
    (d) $e^{-\lambda}$
    (e) $\lambda$

40. Random Walk problem: Define $S = \sum_{i=1}^{N} X_i$ and $X_i, i = 1, \cdots, N$ are two-value i.i.d. (independent identically distributed) random variables, $P(X_i = +1) = p$ and $P(X_i = -1) = 1 - p = q$ then $E[S] =$
    (a) $N$
    (b) $N \times p$
    (c) $N \times q$
    (d) $N(p-q)$
    (e) 0

41. $S_{XX}(\omega) = \int_{-\infty}^{\infty} R_{XX}(\tau) e^{-j\omega\tau} d\tau$ is valid if $X(t)$ is
    (a) first-order stationary
    (b) second-order stationary
    (c) any process with zero mean
    (d) ergodic process
    (e) any process with unit variance

42. Telephone calls are initiated through an exchange at the average rate of 30 per minute and are described by a Poisson process. The probability that less than 3 calls are initiated in any 2-second period =
    (a) $1 - e^{-b}[1 + b + b^2/2]$ with $b = 1$
    (b) $1 - e^{-b}[1 + b]$ with $b = 0.5$
    (c) $e^{-b}[1 + b + b^2/2]$ with $b = 1$
    (d) $e^{-b}[1 + b]$ with $b = 2.0$
    (e) $1 - e^{-b}[1 + b]$ with $b = 1$

 End of exam.