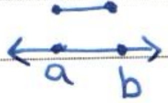
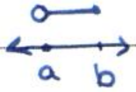
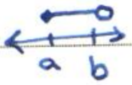
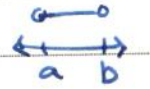
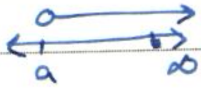
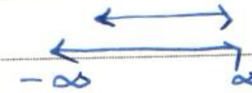
Wed. 28th. sep. 2016

$\boxed{1}$ Intervals الفترات,

	Interval	Descerpition	Graph	ex.
1	$[a,b]$ عدد عناصر هذه الفترة لانهائي	$a \leq x \leq b$		$[1,2]$
2	$[a,b]$ or $]a,b]$	$a < x \leq b$		$(1,2]$ --> (*)
3	$[a,b)$	$a \leq x < b$		$[1,2)$
4	(a,b)	$a < x < b$		$(1,2)$
5	$(-\infty,a]$	$x \leq a$		$(-\infty,1]$
6	$(-\infty,a)$	$x < a$		$(-\infty,1)$
7	$[a,\infty)$	$x \geq a$		$[1,\infty)$
8	(a,∞)	$x > a$		$(1,\infty)$
9	$(-\infty,\infty)$	$\mathbb{R}$		$(-\infty,\infty)$
10	$\{\ \}$	ϕ		$\{\ \}$

2 Solving linear inequalities ::

حل المتباينات الخطية

(equality =) (inequality ≠)

greater than > less than <

greater than or equal ≥ less than or equal ≤

ex:- Solve each of the following inequalities ::

① $4x+5 \leq 6x-9$

Step(1) $4x+5-6x+9 \leq 0$

$-2x+14 \leq 0$

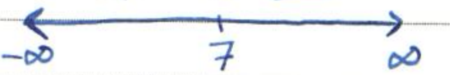
Step(2) find the critical points إيجاد النقاط الحرجة

$-2x+14=0$

$-2x=-14 \longrightarrow \boxed{x=7}$

step(3) Study the sign of the expression.

Sign of $-2x+14$



$[7,\infty)$

Sol expression المقدار ≥ ≤ zero

left hand side
L.H.S
الطرف الأيسر

Right hand side
R.H.S
الطرف الأيمن.

Home-work ← ② $x^2-16>0$

③ $\frac{x-1}{(x-3)(x-5)} < 0$

④ $\frac{1}{x} \geq 1$

Remark

$\leq 0 = -, 0$

$< 0 = -$

$\geq 0 = +, 0$

$> 0 = +$

- Expression = $\frac{\text{بسط}}{\text{مقام}}$ = $\frac{\text{numerator}}{\text{denominator}}$

- Critical point = البسط = صفر

  المقام = صفر

- expression is undefined $\Longrightarrow$ when denominator = 0

③ $\frac{x-1}{(x-3)(x-5)}$

Step(1) ✓

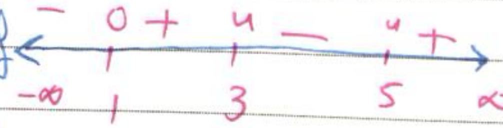
Step(2) Critical point

$x-1=0 \Longrightarrow x=1$

$(x-3)(x-5)=0 \Longrightarrow x=3$

$x=5$

Step(3) Sign of $\frac{x-1}{(x-3)(x-5)}$



Solution set is:-

$(-\infty,1) \underset{\text{union}}{\cup} (3,5)$

اتحاد

④ $\frac{1}{x} \geq 1$

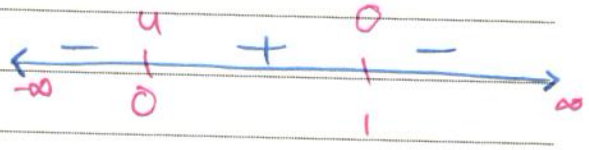
Step (1) $\frac{1}{x} - 1 \geq 0$

Step (2) Critical point

$1 - x = 0 \longrightarrow \boxed{x = 1}$

$\boxed{x = 0}$

Step (3) Sign of $\frac{1-x}{x}$



Solution set $(0, 1]$

***Home-work :** Solve for x :

① $1 + x^2 \geq 0$
② $1 + x^2 < 0$
③ $2 - 3x \leq 2x + 7 < 3x - 1$

Ex: Solve for x :-

① $1 + x^2 \boxed{\geq 0} \longrightarrow (+)$
② $1 + x^2 \boxed{\leq 0} \longrightarrow (-)$
③ $\frac{(x-2)^2}{x-3} \geq 0$
④ $2 - 3x \leq 2x + 7 < 3x - 1$

Sol(1): **Step(1)** ✓

Step(2): Critical points.

$1+x^2 \neq 0$

if $1+x^2=0 \Rightarrow x^2=-1$

$x=\sqrt{-1}$

$\Rightarrow \notin \mathbb{R}$

Real numbers

step(3)

Sign of $1+x^2$

Solution set is $\mathbb{R} = (-\infty, \infty)$

Sol(2):

Solution set is $\{\} = \phi$

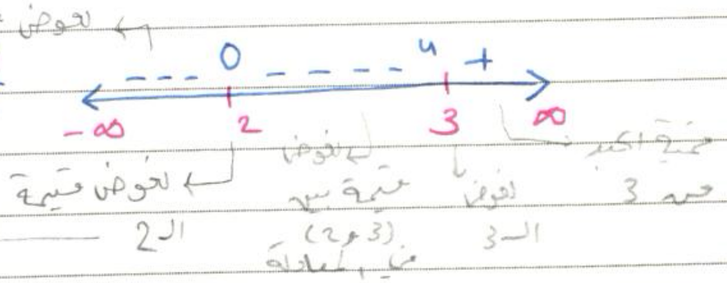
Sol(3): $(x-2)^2=0 \Longrightarrow \boxed{x=2}$

$x-3=0 \Longrightarrow \boxed{x=3}$

$(x-2)^2 \neq (x-2)(x+2)$

$(x-2)^2 = (x-2)(x-2)$

Sign of $\dfrac{(x-2)^2}{x-3}$



$\left(\frac{0}{-1}\right) 0$

$\left(\frac{-1}{0}\right) u$

Solution set $\{2\} \cup (3, \infty)$

Sol (4)

$2-3x \leq 2x+7 < 3x-1$

$2-3x \leq 2x+7 \cap 2x+7 < 3x-1$

خل كل طرف لحال ثم نجد العناصر المشتركة.

* Remark :

$A \leq b \leq c \longrightarrow a \leq b \cap b \leq c$

$a \leq b < c \longrightarrow a \leq b \cap b < c$

$a < b \leq c \longrightarrow a < b \cap b \leq c$

$a < b < c \longrightarrow a < b \cap b < c$

↳ تقاطع inter-Section

(common elements العناصر المشتركة)

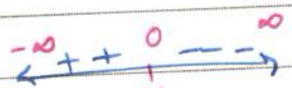
$2-3x-2x-7 \leq 0 \cap 2x+7-3x+1 < 0$

$-5x \quad -5 \leq 0 \quad \cap -x+8 < 0$

$-5x-5=0 \qquad \cap -x+8=0$

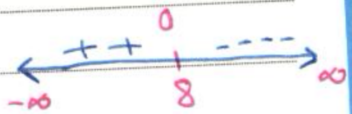
$\Longrightarrow \boxed{x=-1} \qquad \Longrightarrow \boxed{x=8}$

Sign of $-5x-5=0$



$\cap$

Sign of $-x+8$



solution set

$[-1, \infty)$

$\cap$

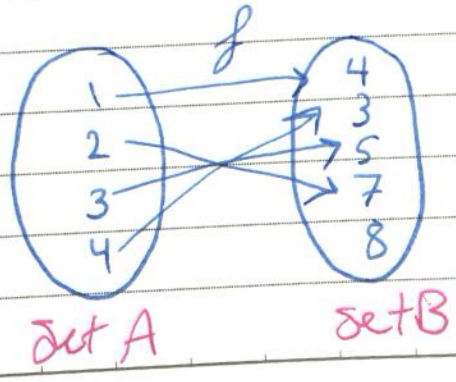
Solution set

$(8, \infty)$

Solution set is $(8, \infty)$

~~Set~~ ③ functions الإقترانات

- $y = f(x)$ ص = ق(س)



* Set A is called (domain) of f (مجال)
* Set B " " (co-domain) of f (مجال مقابل)

تابع $\{3, 4, 5, 7\}$ is called Range of f (مدى)

④ Examples of common function

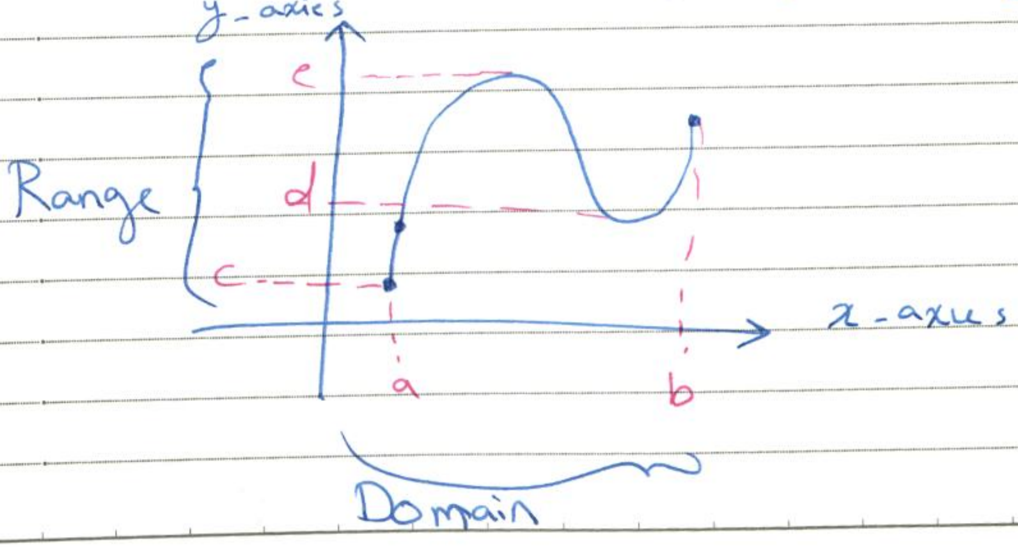
(a) Constant function.

$f(x) = C \rightarrow$ constant

-ex:- $f(x) = 5$, $f(x) = -3$

Domain $(-\infty, \infty) = \mathbb{R}$
Range is $\{C\}$

* Domain and Range graphically.



* Domain :- is starting pt to ending pt

* Range :- is Min to Max

Ex :- find the domain and Range of :-

① $f(x) = 3$
② $f(x) = -1$

Sol (1) Domain of f is $\mathbb{R}$
Range of f is $\{3\}$

Sol (2) Domain of f is $\mathbb{R}$
Range of f is $\{-1\}$

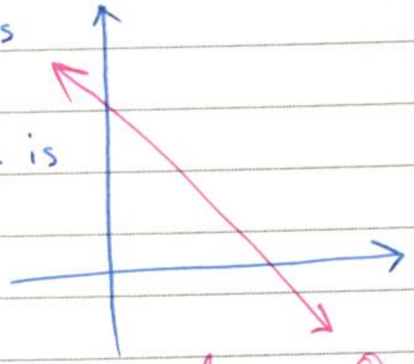
(b) linear function الإقتران الخطي

$f(x) = ax + b$

if $a > 0$

* Domain is $\mathbb{R}$

* Range is $\mathbb{R}$



if $a < 0$

Ex:- find the domain and the range of

① $f(x) = 4-3x$, $a=-3, b=4$

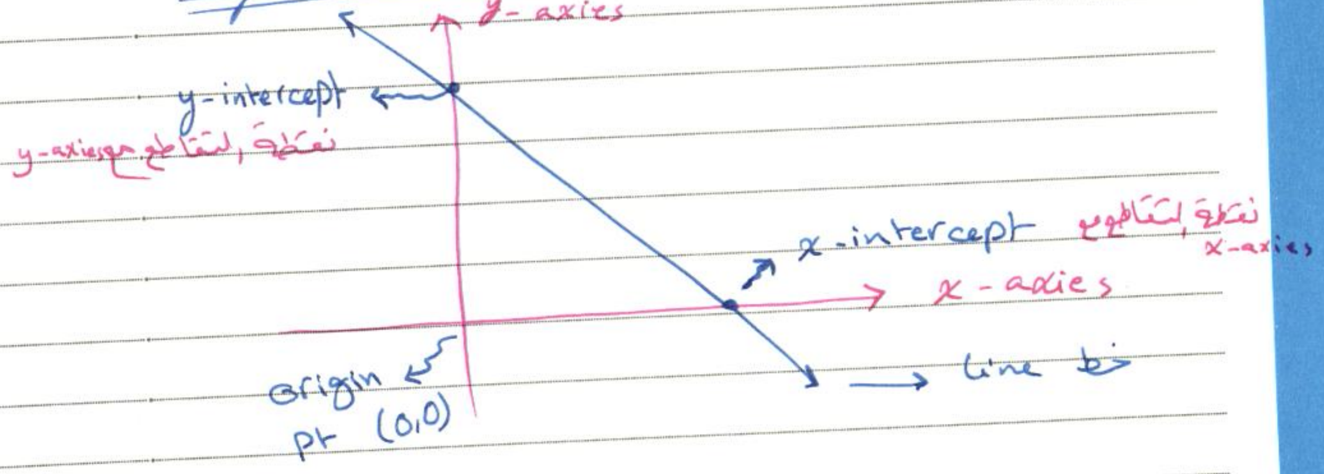
② $f(x) = 5x$, $a=5, b=0$

Sol (1) Domain of $f(x)$ is $\mathbb{R}$

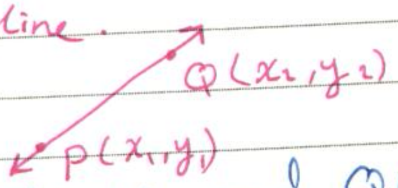
Range of $f(x)$ is $\mathbb{R}$

Sol (2) same solution of sol (1)

* Equation of linear function :-



* Equation of the line.



- Slope of the line that passing through $p(x_1,y_1)$ and $Q(x_2,y_2)$ is $\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

Slope ← $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

* Equation of the line that passing through $p(x_1, y_1)$ and $Q(x_2, y_2)$

$y - y_1 = m\ (x - x_1)$ where.

$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

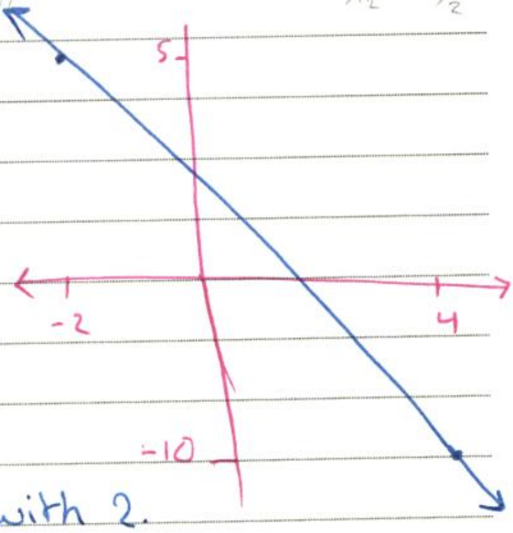
Ex: find the equation of the line that passing through $p(\underset{x_1}{-2}, \underset{y_1}{5})$ and $Q(\underset{x_2}{4}, \underset{y_2}{-10})$

$\boxed{y - 5 = \frac{-5}{2}\ (x + 2)}$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{-10 - 5}{4 + 2}$

$= \frac{-15}{6} = \frac{-5}{2}$



OR :-

multiply both sides with 2.

$2y - 10 = -5x - 10$

$2y - 10 + 5x + 10 = 0$

$\boxed{2y + 5x = 0}$

Wed. 5th. oct. 2016.

Ex:- find the x-intercept and y-intercept of

① $2x+3y=18$

② $\frac{x}{5}+\frac{y}{3}=10$

Sol x-intercept occure when $y=0$
تحدث

$$2x+3y=18$$
$$2x+3(0)=18$$
$$\Rightarrow \boxed{x=9}$$

* the point of intercept is (~~x-intercept~~ x-coordinate, y-coor.)
$=(9,0)$

* The x-intercept is 9

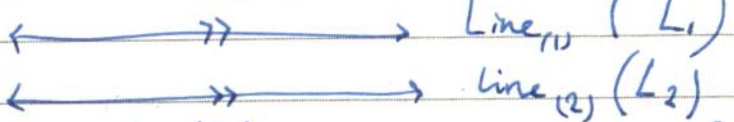
* = y-intercept ~~is 0~~ when $x=0$

$$2x+3y=18$$
$$2x(0)+3y=18$$
$$\boxed{y=6}$$

* The point of y-intercept is (0, 6)

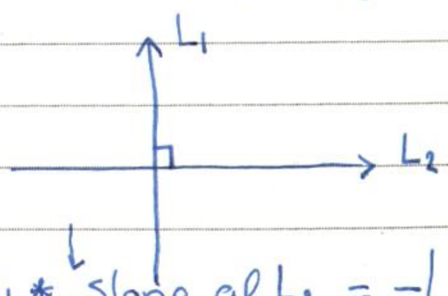
* The y-intercept is 6

- parallel توازي



* $L_1 // L_2 \Leftrightarrow$ slope of L_1 = slope of L_2

- perpendicular



$L_1 \perp L_2 \Leftrightarrow$

slope of L_1 * slope of L_2 = -1

ex. find the equation of the line :

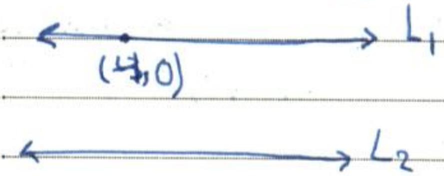
① x intercept 4 and parallel to line $1 - 3x = y$?

② x-intercept -3 and y-intercept $\frac{1}{2}$

③ passes (-3,1) and perpendcular $1 - \frac{x}{2} = y$

$y - y_1 = m(x - x_1)$

Sol(1):



$1 - 3x = y$
slopis -3

Remark

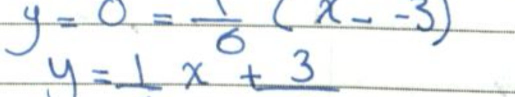
The slope of $\boxed{y = mx + b}$ is m
↓
[illegible] Cofficient of x

Slope of L_1 = slope L_2

So the equation of L_1 is

$$y - y_1 = m(x - x_1)$$
$$y - 0 = -3(x - 4)$$
$$y = -3x + 12$$

Sol(2).



slope = $m = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{\frac{1}{2} - 0}{0 - -3}$
$= \frac{1}{6}$

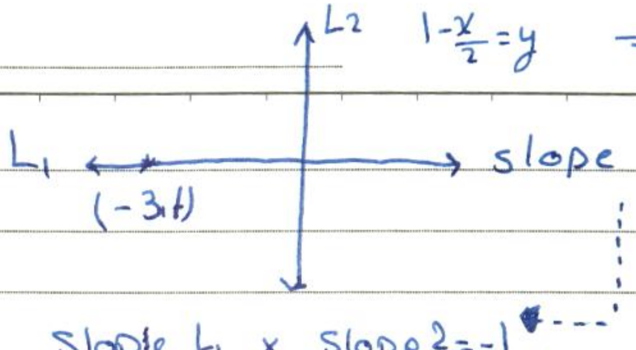
So the equation is

$$y - y_1 = m(x - x_1)$$
$$y - 0 = \frac{1}{6}(x - -3)$$
$$y = \frac{1}{6}x + \frac{3}{6}$$

x معامل
↑

$1 - \frac{x}{2} = y \Rightarrow$ slope of $L_2 = -\frac{1}{2}$

Sol(3)



slope $L_1 = 2$

Slope $L_1 \times$ Slope 2 = -1

Slope $L_1 \times \frac{-1}{2} = -1$

Slop $L_1 = 2$

* The equation of L_1.

$y - 1 = 2(x+3)$

$y - 1 = 2x + 6$

$\boxed{y = 2x + 7}$

☒ let ex: $2x + 3y = 6$ find the slope.

$3y = 6 - 2x$

$y = \frac{6}{3} - \frac{2x}{3}$

slope $= \frac{-2}{3}$ x معامل

Ex: let $2(1-3x) + y = 0$ find
لتكن

① Slope of the line
② x-intercept
③ y-intercept
④ Sketch the line

Sol (1) $2 - 6x + y = 0$

$y = \cancel{2x} + 6x - 2$

slope $= 6$

Sol(2). x-intercept when $y=0$

$y=6x-2$

$0=6x-2$

$\boxed{x=\frac{1}{3}}$

So the x-intercept is $\frac{1}{3}$ $(\frac{1}{3},0$

Sol(3). y-intercept when $x=0$

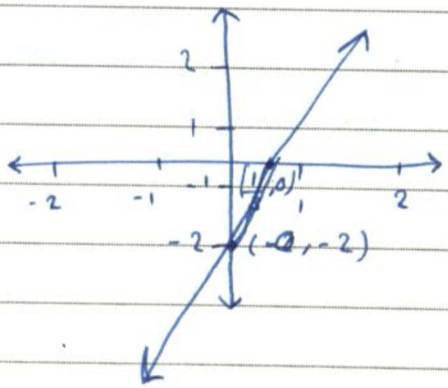
$y=6x-2$

$y=0-2$

$\boxed{y=-2}$

the y-intercept is -2 $(0,-2)$

Sol(4)



* Solving system of linear inequalities graphically.

احتمال كبير
يجي
عليه سؤال حل
٤ علامات

ex: Sketch the solution of the following.

ⓐ $y \leq x$

ⓑ $y < x+5$
$y > 4-2x$

Home work ⓒ $x+y > 1$
$4-3y \leq 0$
$2x+3y < 6$

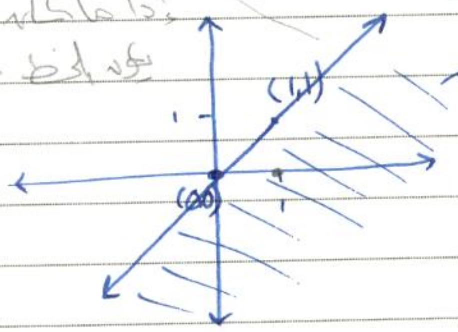
homework ⓓ $x \leq 4$
$y > 2$
$x+y > 6$

sol(a) $y \leq x$; $y = x$.

x	0	0	1
y	0	0	1

(0,0) (1,1)

إذا ما كان في مساواة
يكون الخط متقطع



* نأخذ نقطة من تحت الخط مثل (1,0)
ونعوضها في المعادلة الأصلية

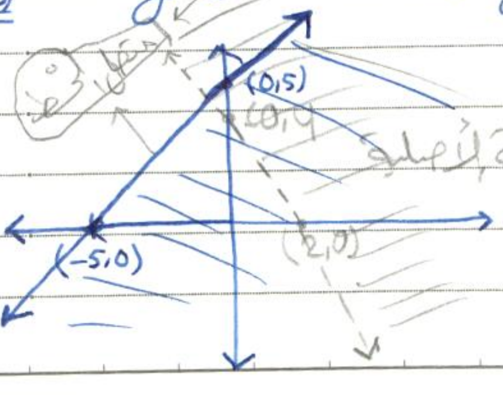
$y \leq x$

Solution set هي المنطقة إلي تحت ⟸ إذاً $0 \leq 1$ ✓✓

Sol(b) $y \leq x+5$; $y = x+5$

x	0	-5
y	5	0

(0,5) (-5,0)



نأخذ نقطة الأصل و نعوضها بالمعادلة الأصلية

$0 \leq 0+5$

$0 \leq 5$ ✓✓

Solution set is under the line

~~مو متأكدة~~

تابع (b) $y \geq 2-2x$; $y = 4-2x$

x	0	2
y	4	0

نرجع هدول الرسمة

(0,4 (2,0)

نعوض (0,0) ← نختار نقطة فوق الخط

$0 \geq 2-0$ او بعيد

$\boxed{X}$

Mon. 10th. oct. 2016

③ Quadratic function (parabola)

الاقتران التربيعي قطع مكافئ

* Domain of $f(x)$ is $\mathbb{R}$

* Range of $f(x)$: if $a > 0$ then the Range of $f(x) = \left[f\left(\frac{-b}{2a}\right)\right]$

if $a < 0$ then the Range of $f(x) = \left(-\infty, f\left(\frac{-b}{2a}\right)\right]$

Ex. Let $f(x) = x^2 - 9$ find.

① Domain and Range of $f(x)$

② vertex

③ x-inter-cept and y-intercept

④ sketch of $f(x)$

Sol (1):- Domain of $f(x)$ is $\mathbb{R}$

Range of $f(x)$ is $\left[f\left(\frac{-b}{2a}\right), \infty\right)$

$$= \left[f\left(\frac{-0}{2(1)}\right), \infty\right)$$

$$= [f(0), \infty)$$

$$= [(0)^2 - 9, \infty)$$

$$= [-9, \infty)$$

Sol (2):- Vertex $= \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

$$= \left(\frac{-0}{2(1)}, f\left(\frac{-0}{2(1)}\right)\right)$$

$$= (0, f(0))$$

$$= (0, -9)$$

Sol (3) x-intercept when $y = 0$

$$y = x^2 - 9 = 0$$

$$(x-3)(x+3) = 0$$

$$\rightarrow \boxed{x = 3} \quad \boxed{x = -3}$$

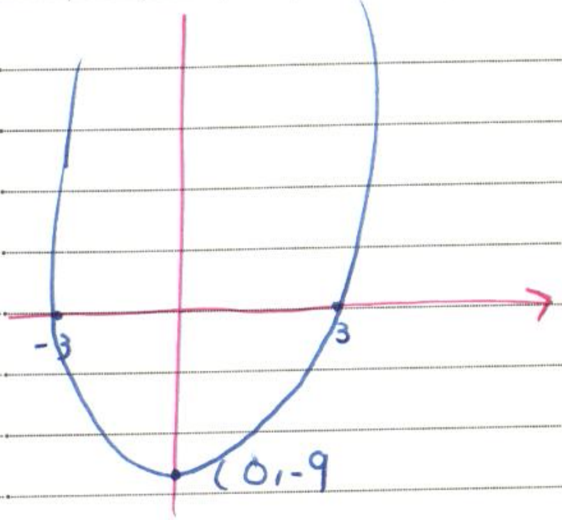
y-intercept when $x = 0$

$$y = x^2 - 9$$

$$y = (0)^2 - 9$$

$$\boxed{y = -9}$$

Sol (4)



xy - plane
or Cartesian
Coordinate
System

Homework :- Repeat the previous example for

① $f(x) = 4x - x^2$

② $f(x) = x^2 + 9$.

Sets : المجموعات

الأعداد الطبيعية .

* $\mathbb{N}$ = Natural numbers $\{1, 2, 3, 4, 5, 6, \ldots\}$

الأعداد الصحيحة .

* $I = \mathbb{Z}$ = Integer numbers

$I = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

* Rational numbers الأعداد النسبية

$$\mathbb{Q} = \left\{ \frac{a}{b} : \text{a and } b \neq 0 \text{ are both integers} \right\}$$

ex:- $\frac{-2}{3}, \frac{1}{2}, 4 = \frac{4}{1}, 0.2 = \frac{2}{10}, 0.\overline{3} = 0.33333$

$= \frac{1}{3}$

* $\mathbb{R} = (-\infty, \infty)$ = Real numbers . الأعداد الحقيقية

$\mathbb{N} \subseteq I \subseteq \mathbb{Q} \subseteq \mathbb{R}$

↓
subset
احتواء .

Ex:- $\frac{-2}{3} \in \mathbb{Q}$ $\boxed{\checkmark}$

↳ belongs to / in / تنتمي إلى

$4 \in \mathbb{R}$ $\boxed{\checkmark}$

$\frac{3}{4} \in \mathbb{N}$ $\boxed{\times}$

* $\{1,2\} \subseteq \{1,2,3,4,5\}$ $\boxed{\checkmark}$

* $1 \subseteq \{1, 2, 3\}$ $\boxed{\times}$

ex.:- Use the listing methods to descibe the following sets:-

① The set of all integers less than 5 and greater than -2.

$\{-1, 0, 1, 2, 3, 4\}$

② " " " " Natural numbers less than 50.

$\{1, 2, 3, \dots, 49\}$

الأعداد الأكبر من ١ ، ولا تقبل القسمة إلا على نفسها او على $\boxed{1}$

③ " " " " Prime numbers less than 20

الأعداد الأولية

$\{2, 3, 5, 7, 11, 13, 17, 19\}$

④ " " " " even numbers less than 100.

الأعداد الزوجية

$\{\dots\dots, -4, -2, 0, 2, 4, \dots\dots, 98\}$

even numbers

$\{ \ldots, -4, -2, 0, 2, 4 \ldots \}$

odd numbers

$\{ \ldots, -5, -3, -1, 1, 3, 5, 7, 9, \ldots \}$

positive even numbers.

$\{ 2, 4, 6, 8 \ldots \}$

ex:: which of the following statement are True and which are false.

① $2 \in \{1,2,3\}$ True.

② $3 \subseteq \{1,2,3,4\}$ false.

③ $\{5\} \subseteq \{1,2,3,5,7\}$ True.

Subset

لازم يكون قبلها مجموعة

④ The Rational function الإقتران النسبي

$f(x) = \frac{p(x)}{q(x)}$
where $p(x)$ and $q(x) \neq 0$ are both polynomial
كسر كبير

Special case:- $f(x) = \frac{ax+b}{cx+d}$

* Domain of $f(x)$ is
$\mathbb{R} - \{\frac{-d}{c}\}$

* Range of $f(x)$ is
معامل البسط
$\mathbb{R} - \{$ coefficient of x nu
″ = x for domem
معامل المقام
$\mathbb{R} - \{\frac{a}{c}\}$

ex: Determain the domain and the Range for

① $f(x) = \frac{2x+4}{3x-6}$

② $f(x) = \frac{4x}{x-2}$

③ $f(x) = \frac{2}{x+1} = \frac{0x+2}{x+1}$

Sol(1) * Domain is $\mathbb{R} - \{\frac{6}{3}\}$

$\mathbb{R} - \{2\}$

* Range. $\mathbb{R} - \{\frac{2}{3}\}$

Sol(2)

* Domain $\mathbb{R} - \{\frac{2}{1}\}$

$\mathbb{R} - \{2\}$

* Range $\mathbb{R} - \{\frac{4}{1}\} \Rightarrow \mathbb{R} - \{4\}$

Sol(3)

* Domain $\mathbb{R}\ \{-1\}$

* Range $\mathbb{R} - \{0\}$

⑤ The n^{th} root function الإقتران الجذر النوني

$$\boxed{f(x) = \sqrt[n]{g(x)}}$$

* Domain $f(x) = \{x : g(x) \geq 0$ if n is even$\}$

* Domain of $f(x)$ = Domain of $g(x)$ if n is odd

Ex:- find the domain of the following.

① $f(x) = \sqrt{x^2-4}$

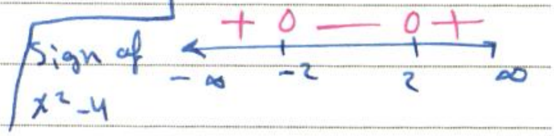
② $f(x) = \sqrt[3]{\dfrac{x-1}{x+4}}$

③ $f(x) = \sqrt{\dfrac{x-1}{(x-3)(x-5)}}$

Sol (1) $f(x) = \sqrt{x^2-4}$; $x^2-4 \geq 0$

Solving linear inequalities.

* Domain is $(-\infty,-2] \cup [2,\infty)$ $x^2-4=0 \Rightarrow x = -2, 2$



Sol (2) Domain of $f(x)$ = Domain of $\left\{\dfrac{x-1}{x+4}\right\}$

$= \mathbb{R} - \{-4\}$

Wed. 12th. oct. 2016

* Applications (quadratic funktion)

Ex:- find the two numbers x and y that statisfy the condition that $3x - y = 18$ and for which their product xy is as small as possible.

ضرب

Sol:- $p = xy$, but $3x - y = 18$

$p = x(3x - 18)$ $\Rightarrow \boxed{y = 3x - 18}$

$\boxed{p = 3x^2 - 18x}$ quadratic function with $a = 3$, $b = -18$

$$\text{vertex} = \left(\frac{-b}{2a}, p\left(\frac{-b}{2a}\right)\right) = \left(\frac{-(-18)}{2\times3}, p\left(\frac{-(-18)}{2\times3}\right)\right)$$

$$= (3, p(3))$$

~~$p = 3x^2 - 18x$~~

~~$-27 = 3(3)^2 - 18$~~

$$= (3, 3(3^2) - (18\times3))$$

$$= (3, -27)$$

قيمة x قيمة p

$\boxed{x = 3}$ $\boxed{p = -27}$

$y = 3x - 18$

$= 3(3) - 18$

$\boxed{y = -9}$

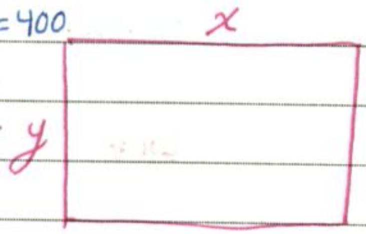
yard

Ex:- A farmer has 400 yd of fencing with which to bulid a ~~recta~~ rectangular field, what is Largest area he can enclose?

Sol $A = xy$, but $2x+2y=400$

$x+y=200$

$\Rightarrow \boxed{y=200-x}$



$A(x) = x(200-x)$

$A(x) = 200x - x^2$ quadratic function with $a=-1$, $b=200$

$$\text{Vertex} = \left(\frac{-b}{2a}, A\left(\frac{-b}{2a}\right)\right)$$

$$= \left(\frac{-200}{2(-1)}, A\left(\frac{-200}{2(-1)}\right)\right) = (100, A(100)$$

$$= (100, 10000)$$

$\boxed{x=100} \leftarrow$ $\boxed{A=10000}$

$y = 200 - x$

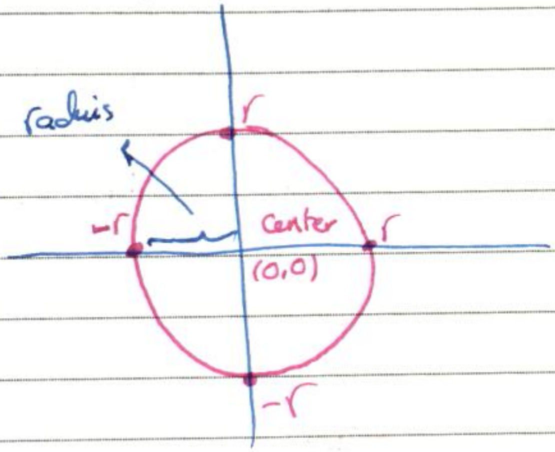
$= 200 - 100$

$\boxed{y = 100}$

⑥ Circle

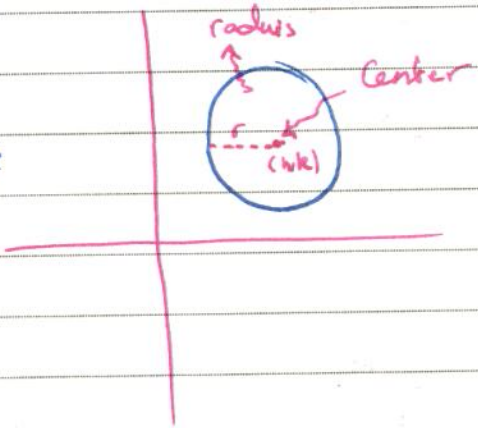
* Equation:-

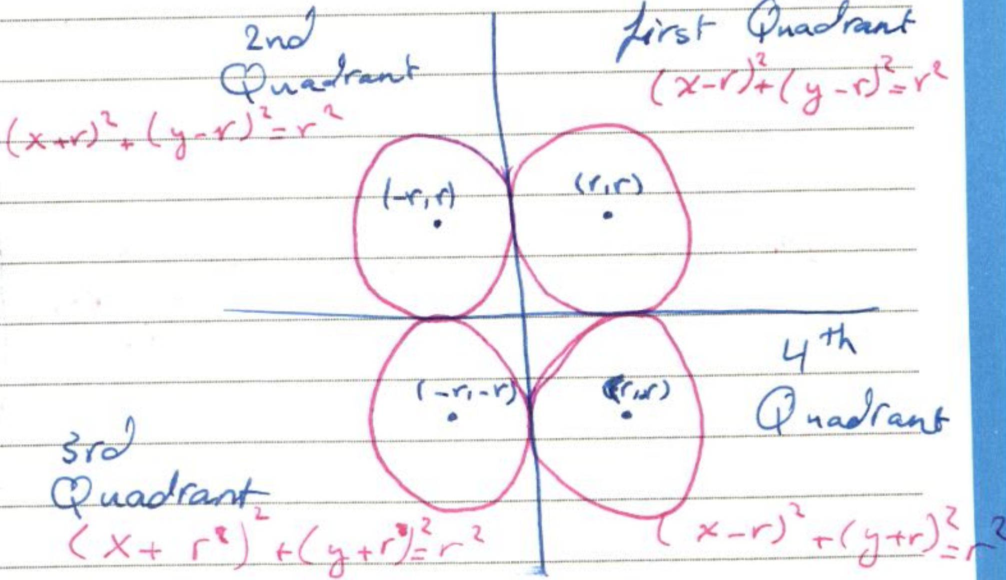
$x^2+y^2=r^2$



* Equation:-

$(x-h)^2+(y-k)^2=r^2$





Ex:- find the equation of the circle whose radii and centers are given

① radius = 5 , Center = (0, -2)
② radius = 4 , center = (-3, 0)
③ radius = 2 , center (0, 0)

Sol (1) $(x-0)^2+(y+2)^2=25 \Rightarrow x^2+(y+2)^2=25$

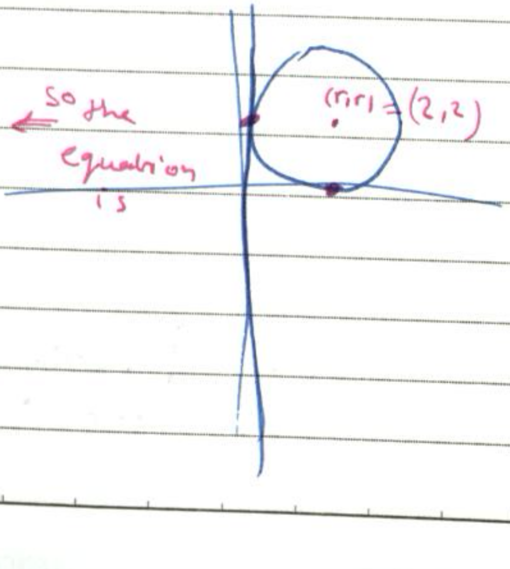
Sol (2) $(x+3)^2+y^2=16$

Sol (3) $x^2+y^2=4.$

Ex:- find the equation of the circle that تقع lies in the first quadrant, has raduis of 2 unites and touches the coordinate axies

(y, x) محور

Sol. $(x-2)^2+(y-2)^2=4 \Leftarrow$ so the equation is



Ex:- find the center and the radius of the following circles:

① $y^2 + (x-3)^2 = 7$
② $x^2 + y^2 - 4 = 0$
③ $x^2 + y^2 - 4x + 6y - 11 = 0$

Sol(1) Center $(3,0)$, radius $= \sqrt{7}$

Sol(2) Center $(0,0)$, radius $= 2.$

Sol(3) Completing square اكمال مربع

$$\left[x^2 - 4x + \left(\frac{\overset{-4}{x\ \text{معامل}}}{2}\right)^2 - \left(\frac{\overset{-4}{x\ \text{معامل}}}{2}\right)^2\right] + \left[y^2 + 6y + \left(\frac{\overset{6}{\text{معامل}}}{2}\right)^2 - \left(\frac{\overset{6}{\text{معامل}}}{2}\right)\right] = 11$$

$$\Rightarrow \left[\underbrace{x^2 - 4x + 4}_{(x-2)^2} - 4\right] + \left[\underbrace{y^2 + 6y + 9}_{(y+3)^2} - 9\right] = 11$$

$$(x-2)^2 + (y+3)^2 = 11 + 4 + 9$$

$$(x-2)^2 + (y+3)^2 = 24$$

$\longrightarrow$ Center $(2, -3)$, radius $= \sqrt{24}$

للسؤال الرابع ٣
إذا كانت
multipl. choice

*Remark :-

for $\boxed{x^2 + ax + y^2 + by + C = 0}$

* then the center $= \left(\frac{-a}{2}, \frac{-b}{2}\right)$

* radius $= \sqrt{\left(\frac{-a}{2}\right)^2 + \left(\frac{-b}{2}\right)^2 - c}$

Ex:- find the radius and the center of $x^2 + y^2 - 4x + 6y - 11 = 0$

$$\text{center} = \left(\frac{-(-4)}{2}, \frac{-6}{2}\right)$$

$$= (2, -3)$$

$$\text{radius} = \sqrt{(2)^2 + (3)^2 - (-11)}$$

$$= \sqrt{4 + 9 + 11}$$

$$= \sqrt{13 + 11}$$

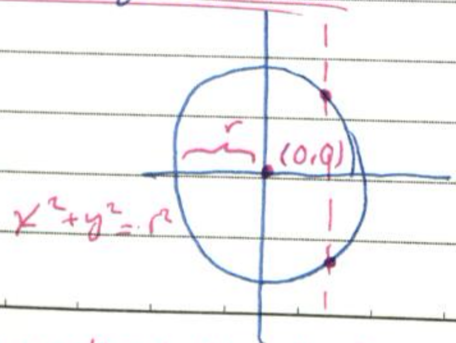
$$= \sqrt{24}$$

* ? Is the circle function.

* garaphically :-

Vertical line test

اختبار الخط العمودي



No by the vertical line test

* gebraically (جبرياً)

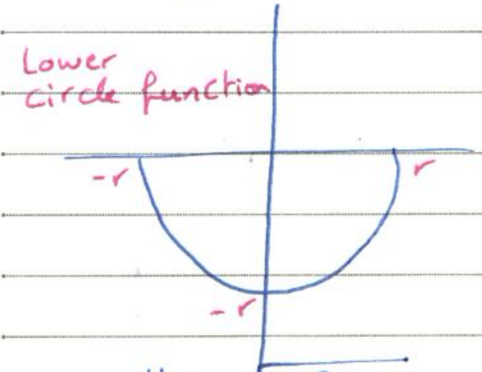
$$x^2+y^2=r^2 \Rightarrow y^2=r^2-x^2 \Rightarrow y=\mp\sqrt{r^2-x^2}$$

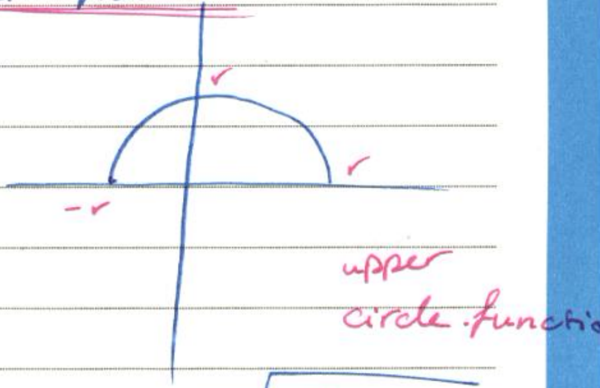
$$y=-\sqrt{r^2-x^2}$$

$$y=+\sqrt{r^2-x^2}$$

$\boxed{\text{So}}$ $x^2+y^2=r^2$ is not function.

* upper circle / Lower function





- $y=-\sqrt{r^2-x^2}$
- Domain is $[-r,r]$
- Range is [~~0,r~~] -r,0

- $y=\sqrt{r^2-x^2}$
- Domain = $[-r,r]$
- Range is $[0,r]$

Mon. 17th oct. 2016

*upper / Lower circle function

نصف بياني باستعمال دائرة

Ex: Sketch and find the domain and the range of the following functions:-

① $y = \sqrt{9 - x^2}$

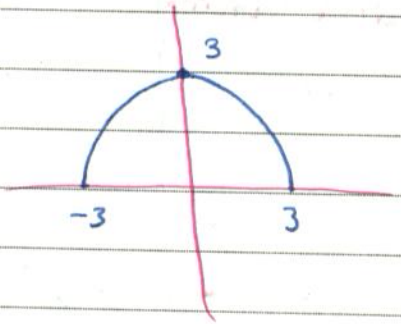
② $y = -\sqrt{16 - x^2}$

③ $y = 5 + \sqrt{4 + x^2}$

④ $f(x) = -3 - \sqrt{1 - x^2}$

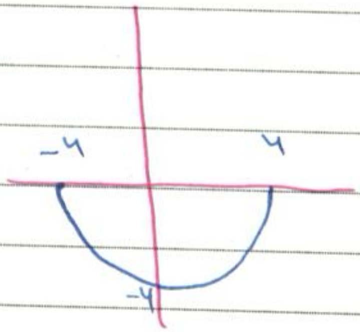
Sol (1).

Domain = $[-3,3]$
Range = $[0,3]$



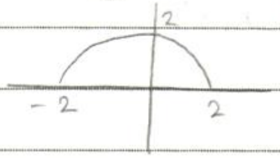
Sol (2).

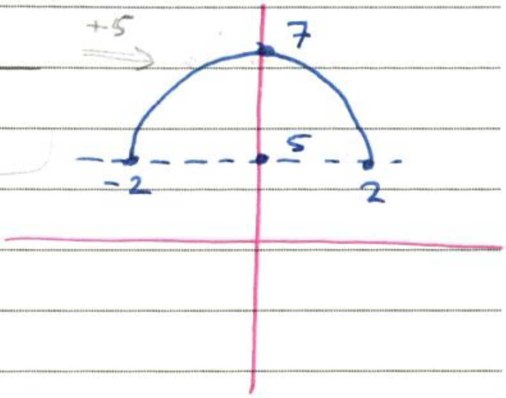
Domain = $[-4,4]$
Range = $[-4,0]$



Sol(3) " Shifting " كل الإزاحة و الإنسحاب

$\sqrt{4-x^2}$

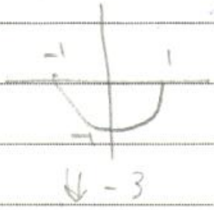




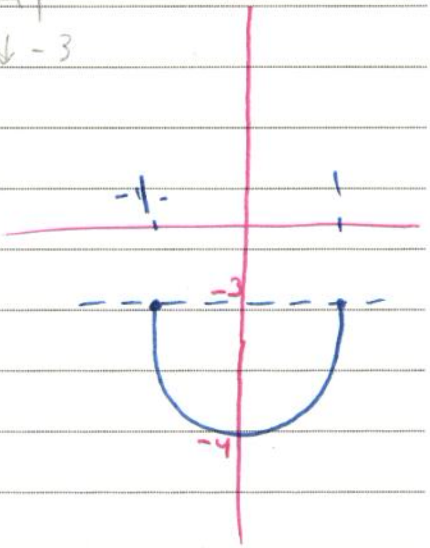
نضيف الخمسة على ال y-axis

Domain = [-2,2]
Range = [5,7]

Sol(4) $-\sqrt{1-x^2}$ ⟹

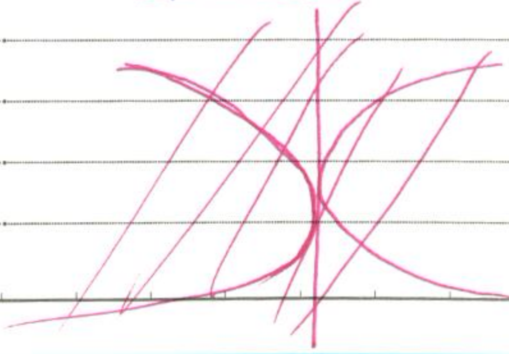


Domain = [-1,1]
Range = [-4, -3]



⑦ The square root function of linear expression :-

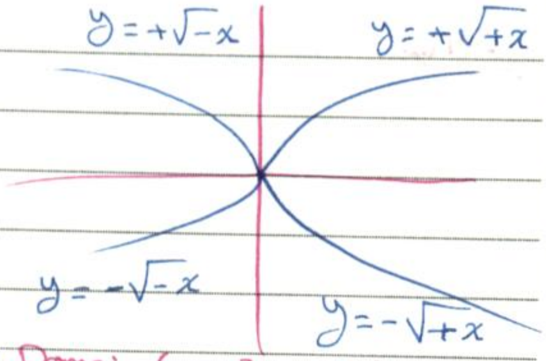
$f(x) = a+\sqrt{bx+c}$ or
$f(x) = a-\sqrt{bx+c}$



* Graph is apart af c letter toward the x-axius

* Domain$(-\infty,0]$ Range$[0,\infty)$

* Domain$[0,\infty]$ Range$=[0,\infty)$



* Domain$=(-\infty,0]$ Range $=(-\infty,0]$

* Domain$[0,\infty)$ Range$(-\infty,0]$

<u>Ex:-</u> Sketch and find the domain and the range af the following ~~equation~~:- function:-

① $y=\sqrt{x-1}$ Starting pt $(1,0)$

Solution

* By starting point

② $y=\sqrt{x+1}$ St. pt $(-1,0)$

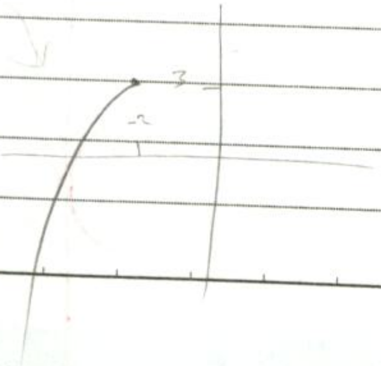
③ $y=-\sqrt{-x+1}$ St. pt. $(1,0)$

④ $y=3+\sqrt{2-x}$ St. pt $(2,3)$

⑤ $y=-3-\sqrt{x+4}$ St. pt $(-4,-3)$

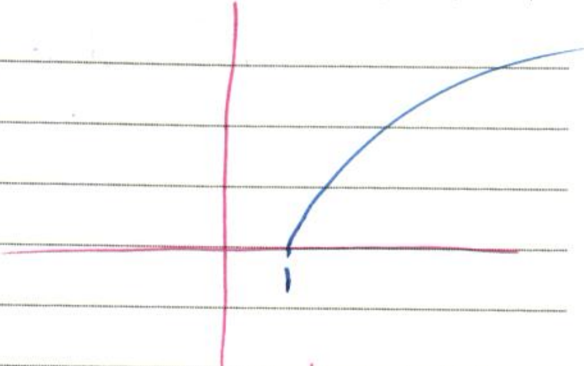
~~6~~

⑥ $y=-2+\sqrt{6-2x}$ St. pt $(3,-2)$



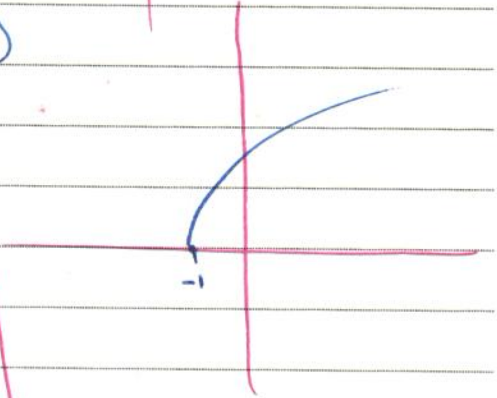
Sol(1) Starting pt $(1,0)$

Domain $= [1, \infty)$
Range $= [0, \infty)$

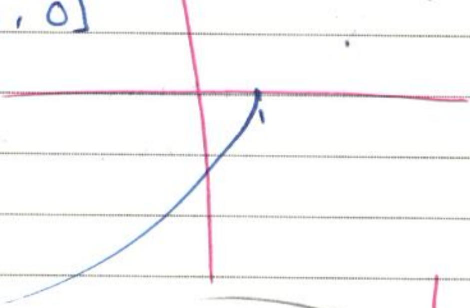


Sol(2)

Domain $= [-1, \infty)$
Range $= [0, \infty)$

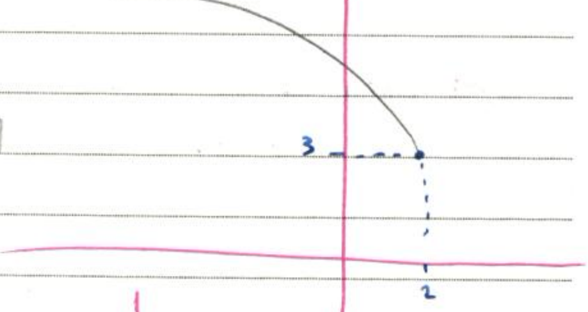


Sol(3) Domain $= (-\infty, 1]$
Range $= (-\infty, 0]$

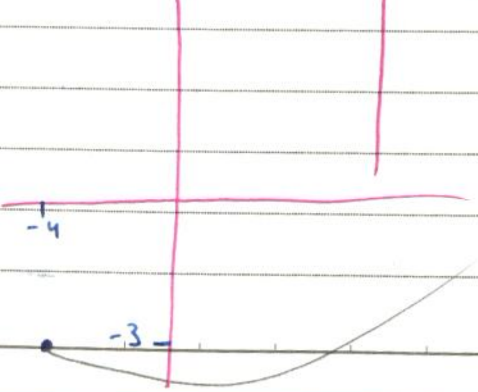


Sol (4)

Domain $= (-\infty, 2]$
Range $= [3, \infty)$



Sol(5) Domain $= [-4, \infty)$
Range $= [-\infty, -3]$



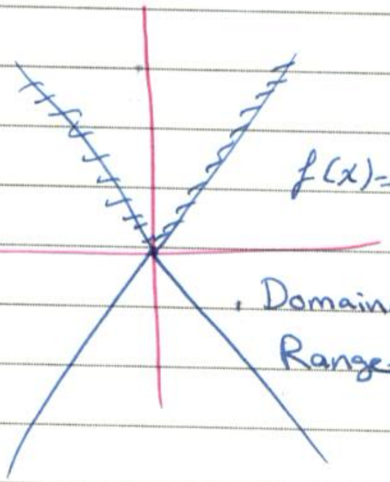
⑧ The absolute value function:-

اقتران ذو القيمة المطلقة.

$$f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

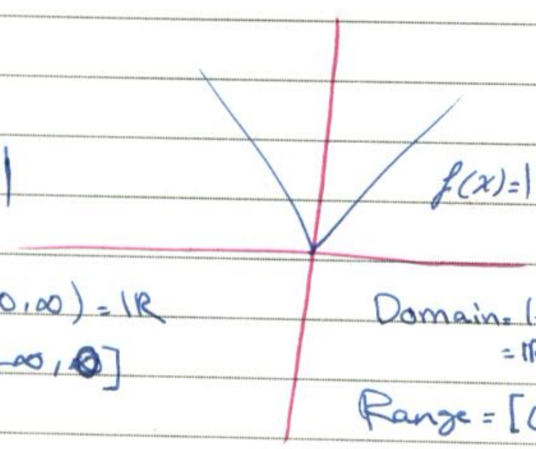
هذه الصيغة: اقتران متشعب/متفرع.

↳ Pice/wise function.



Domain = $(-\infty, \infty) = \mathbb{R}$

Range = $(-\infty, 0]$



Domain = $(-\infty, \infty) = \mathbb{R}$

Range = $[0, \infty)$

Ex: Sketch and find the domain + Range of the following function:- (by st. pt)

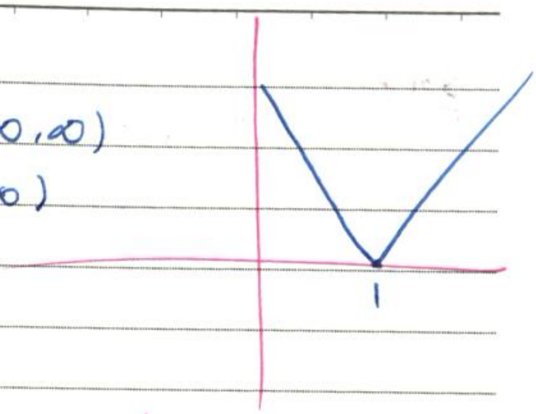
الإشي اللي بخليه صفر 0

① $f(x) = |x-1|$ st.pt. $(1, 0)$
② $f(x) = |x-3|$ st.pt. $(3, 0)$
③ $f(x) = 4 + |2x-4|$ st.pt $(2, 4)$
④ $f(x) = -3 - |5-x|$ st.pt $(5, -3)$
⑤ $f(x) = |x| + 4$ st.pt $(0, 4)$
⑥ $f(x) = 6 - |x|$ st.pt $(0, 6)$

Sol(1)

Domain = $(-\infty, \infty)$

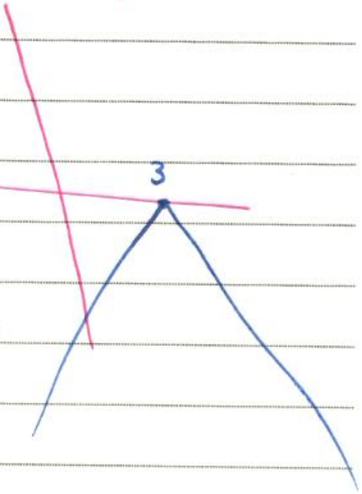
Range = $[0, \infty)$



Sol(2)

Domain = $(-\infty, \infty)$

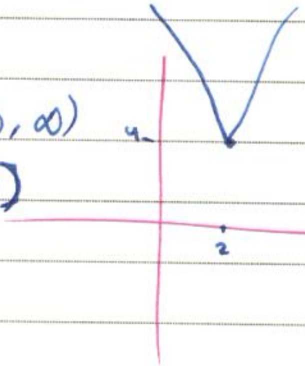
Range = $(-\infty, 0]$



Sol(3)

Domain = $(-\infty, \infty)$

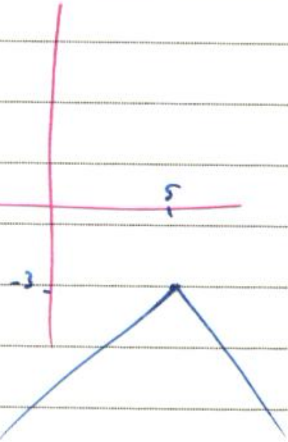
Range = $[4, \infty)$

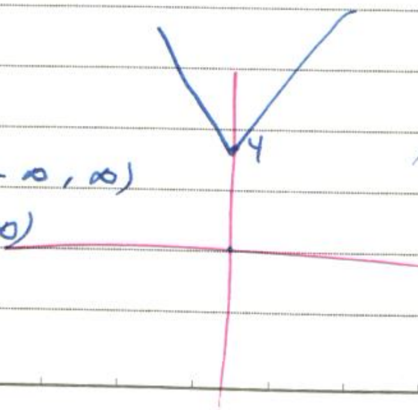


Sol(4)

Domain = $(-\infty, \infty)$

Range = $(-\infty, -3]$





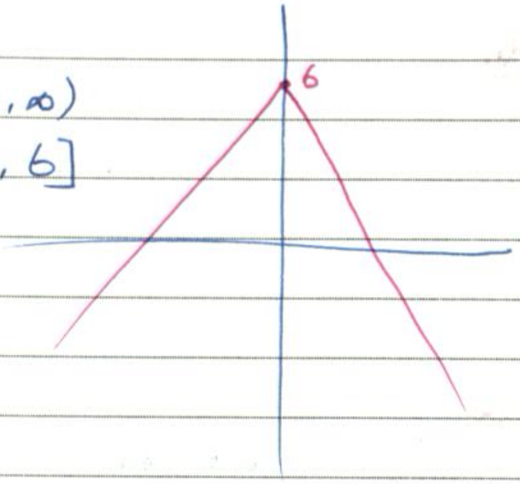
Domain = $(-\infty, \infty)$

Sol(5) Range = $[4, \infty)$

Sol (6).

Domain = $(-\infty, \infty)$

Range $(-\infty, 6]$



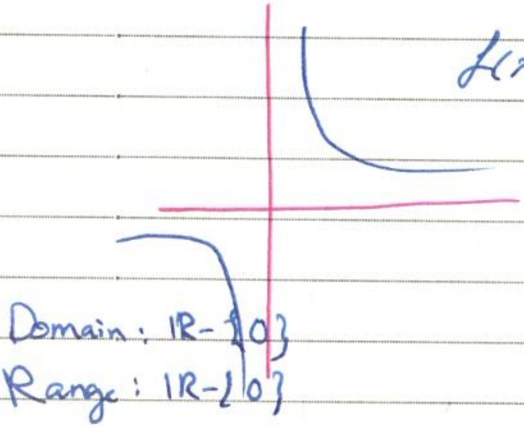
Wed. 19th. oct .2016

⑨ The Rational function of the form:

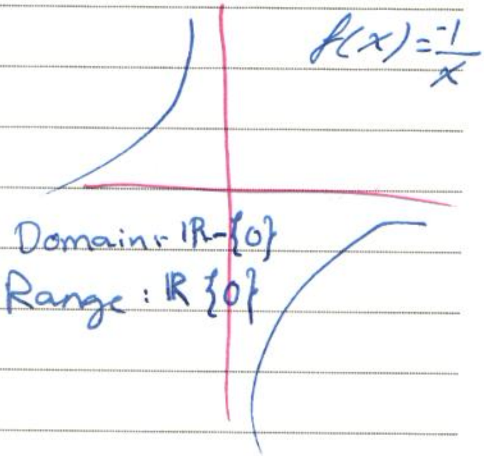
$$f(x) = \frac{a}{bx+c} \quad \boxed{\text{or}} \quad f(x) = \frac{-a}{bx+c} \;;\; b \geq 0$$

* Domain : $\mathbb{R} - \left\{\frac{-c}{b}\right\}$

* Range : $\mathbb{R} - \{0\}$



Domain : $\mathbb{R} - \{0\}$
Range : $\mathbb{R} - \{0\}$



Domain : $\mathbb{R} - \{0\}$
Range : $\mathbb{R} \{0\}$

Ex:- Sketch and find the domain and the Range of the following function.

↙ virtical Line الخط الطولي

① $f(x) = \frac{1}{x-1}$; VL : $x = 1$

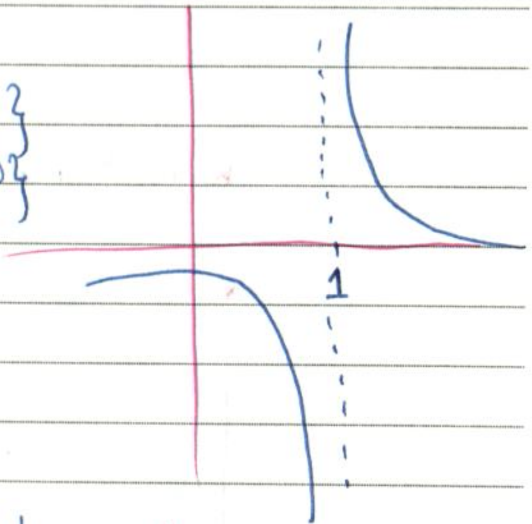
② $f(x) = \frac{-1}{x+4}$; VL : $x = -4$

③ $f(x) = \frac{3}{4-x}$; VL : $x = 4$

④ $f(x) = \frac{-2}{4-2x}$; VL : $x=2$

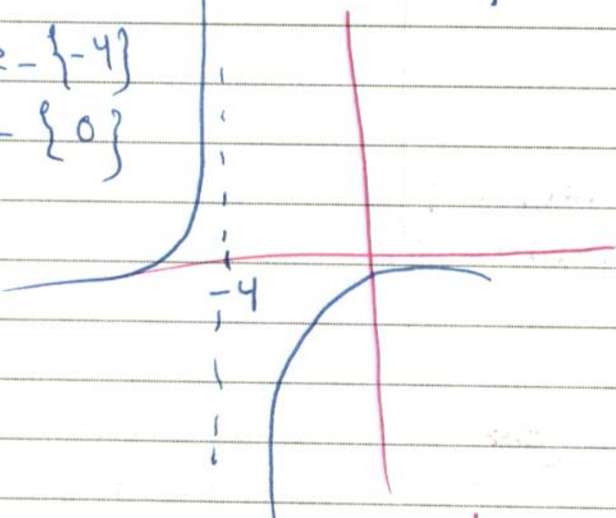
Sol(1) Domain $\mathbb{R} - \{1\}$

Range $\mathbb{R} - \{0\}$



Sol(2) Domain $\mathbb{R} - \{-4\}$

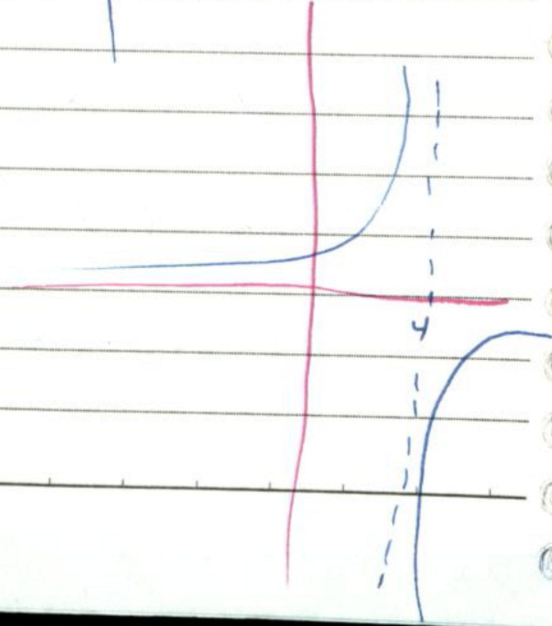
Range $\mathbb{R} - \{0\}$



Sol(3) $f(x) = \frac{3}{4-x} = \frac{-3}{x-4}$

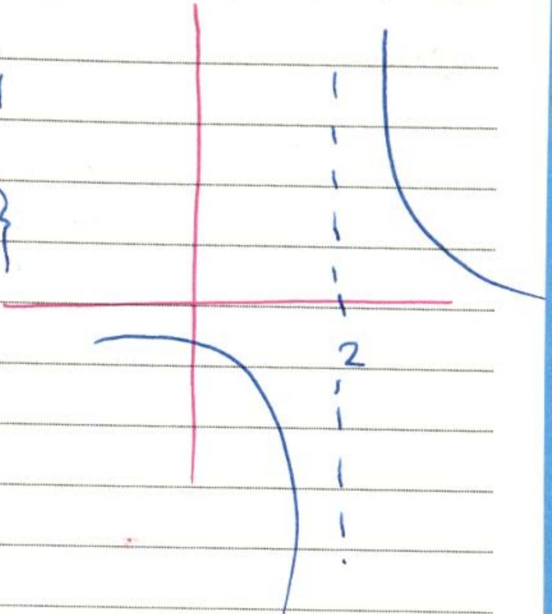
Domain $\mathbb{R} - \{4\}$

Range $\mathbb{R} - \{0\}$



sol(4) $f(x) = \frac{-2}{4-2x} = \frac{2}{2x-4}$

Domain $\mathbb{R} - \{2\}$
Range $\mathbb{R} - \{0\}$



⑩ More functions :-

① Domain $\{f(x) \overset{\times}{\pm} g(x)\}$ = Domain $\{f(x)\}$ $\cap$ Domain $\{g(x)\}$

② Domain $\left\{\frac{f(x)}{g(x)}\right\}$ = Domain $\{f(x)\}$ $\cap$ Domain $\{g(x)\}$ - $\{x : g(x) = 0\}$

اصفار المقام

Ex:- find the domain for each of the following:-

① $f(x) = \sqrt{x-1} + \sqrt{9-x^2}$

② $f(x) = \frac{\sqrt{x^2-16}}{x^2-25}$

③ $f(x) = \frac{x}{x-2} - \sqrt{x^2-4x}$

④ $f(x) = (\sqrt[5]{x^2+4})(\sqrt{x})$

Sol (1) $f(x) = \sqrt{x-1} + \sqrt{9-x^2}$

$\text{Domain}\{f(x)\} = \text{Domain}\{\sqrt{x-1}\} \cap \text{Domain}\{\sqrt{9-x^2}\}$

$x-1 \geq 0$ $= [-3,3]$

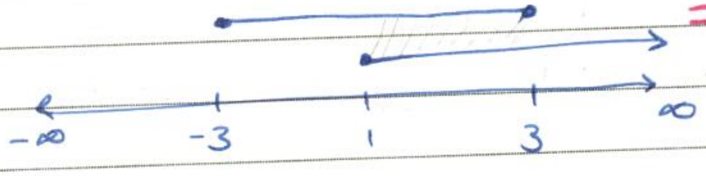
"upper circle"

Sign of $x-1$: $\longleftarrow - - - - \overset{0}{\phi} + + + \longrightarrow$

$\text{Domain}\{\sqrt{x-1}\} = [1,\infty)$

* $\text{Domain } f(x) = [1,\infty) \cap [-3,3]$

$= [1,3]$



Sol (2) $\text{Domain } f(x) = \text{Domain}\{\sqrt{x^2-16}\} \cap \text{Domain}\{x^2-25\}$

$- \{x : x^2-25=0\}$

* Domain $\mathbb{R}$

$x^2-16 \geq 0$

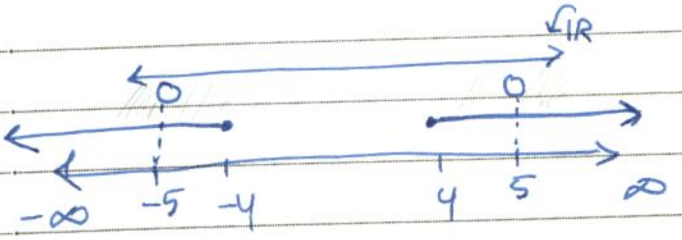
$\boxed{x=4}$, $\boxed{x=-4}$

Sign of x^2-16: $-\infty \quad + \quad \overset{0}{-4} \quad - \quad \overset{0}{+4} \quad + \quad \infty$

* Domain $(-\infty,-4] \cup [4,\infty)$

$x^2-25=0$

$\Longrightarrow \boxed{x=-5,5}$



$\text{Domain } f(x) = (-\infty,-5) \cup (-5,-4] \cup [4,5)$
$\cup (5,\infty)$

$(-\infty,4] \cup [4,\infty) - \{-5,5\}$ $\boxed{\text{or}}$

$(-\infty,-4] - \{-5\} \cup [4,\infty) - \{5\}$ $\boxed{\text{or}}$

⑪ Composition function تركيب الإقترانات

(و ∘ هـ) (س) = و (هـ(س))

$(f \circ g)(x) = f(g(x))$

بعد التركيب

* Domain of $(f \circ g)(x)$ = Domain of $f(g(x)) \cap$ Domain of $g(x)$ الإقتران الداخلي

* Domain of $(g \circ f)$ = Domain of $g(f(x)) \cap$ Domain of $f(x)$

Ex:- Given $f(x) = x^2$ and $g(x) = \sqrt{x-1}$

evaluate :-

① $(f \circ g)(5)$ ② $(g \circ f)(\frac{4}{3})$

Sol(1) $f(g(5)) = f(\sqrt{5-1}) = f(2) = (2)^2 = 4$

Sol(2) $(g \circ f)(\frac{4}{3}) = g(f(\frac{4}{3})) = g((\frac{4}{3})^2)$

$= g(9) = \sqrt{\frac{9}{9} - 1}$

$= \sqrt{\frac{8}{[illegible]}}$

$=$

Ex:- Determain $(f \circ g)(x)$ and $(g \circ f(x)$ and find domain of $f \circ g$ and $g \circ f$:-

① $f(x) = x^2$, $g(x) = 1 + x$
② $f(x) = \sqrt{x+1}$, $g(x) = x^2$
③ $f(x) = \frac{1}{x+1}$, $g(x) = \sqrt{x} + 1$
④ $f(x) = \frac{1 \;\; x+1}{x-1}$, $g(x) = \frac{1}{x-2}$.

Sol(1). * $(f \circ g)(x) = f(g(x)) = f(1+x) = (1+x)^2$

$= x^2 + 2x + 1$

* $(g \circ f)(x) = g(f(x)) = g(x^2) = 1 + x^2$

* Domain $(f \circ g)$ = Domain $f(g(x)) \cap$ Domain $g(x)$

= Domain $(x^2 + 2x + 1) \cap$ Domain $(1+x)$

$\mathbb{R} \cap \mathbb{R} = \mathbb{R}$

* Domain $(g \circ f)$ = Domain $(g(f(x))) \cap$ Domain $(f(x))$

= Domain $(1 + x^2) \cap$ Domain (x^2)

$\mathbb{R} \cap \mathbb{R} = \mathbb{R}$.

Sol(4). * $(f \circ g)(x) = f(g(x) = f\left(\frac{1}{x-2}\right) = \frac{1}{\frac{1}{x-2} - 1}$

$$= \frac{1}{\frac{1}{x-2} - \frac{x-2}{x-2}}$$

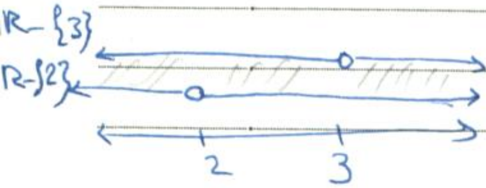
$$= \frac{1}{\frac{1-(x-2}{x-2}}$$

$$= \frac{x-2}{1-x+2}$$

$$= \frac{x-2}{3-x}$$

* Domain $(f \circ g)$ = Domain$(f(g(x))) \cap$ Domain$(g(x))$

= Domain$\left(\frac{x-2}{3-x}\right) \cap$ Domain$\left(\frac{1}{x-2}\right)$



$= \mathbb{R} - \{3\} \cap \mathbb{R} - \{2\}$

$= \mathbb{R} - \{2, 3\}$

$$* \quad (g \circ f)(x) = g(f(x)) = g\left(\frac{1}{x-1}\right) = \frac{1}{\frac{1}{x-1} - 2}$$

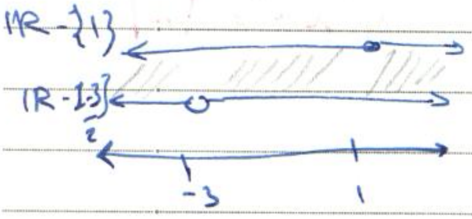
$$= \frac{1}{\frac{1}{x-1} - \frac{2(x-1)}{x-1}}$$

$$= \frac{1}{\frac{1-2x-2}{x-1}}$$

$$= \frac{1}{\frac{-3-2x}{x-1}}$$

$$= \frac{x-1}{-3-2x}$$

$$* \quad \text{Domain}(g \circ f) = \text{Domain}(g(f(x)) \cap \text{Domain}(f(x))$$



$$= \text{Domain}\left(\frac{x-1}{-3-2x}\right) \cap \text{Domain}\left(\frac{1}{x-1}\right)$$

$$\mathbb{R} - \left\{\frac{-3}{2}\right\} \cap \mathbb{R} - \{1\}$$

$$\mathbb{R} - \left\{1, -\frac{3}{2}\right\}$$

ex:- find $g(x)$ if

(1) $f(x) = x^2$ and $(f \circ g)(x) = (1+x)^2$

(2) $f(x) = \frac{x+1}{x+2}$ and $(g \circ f)(x) = \frac{3}{x-2}$

Sol(1):-

$$(f \circ g)(x) = (1+x)^2$$
$$f(g(x)) = (1+x)^2$$
$$(g(x))^2 = (1+x)^2$$
$$\sqrt{(g(x))^2} = \sqrt{(1+x)^2}$$
$$|g(x)| = |1+x|$$
$$\Rightarrow g(x) = 1+x \quad \text{or} \quad g(x) = -(1+x)$$

Remark

$(\sqrt{x})^2 = x$

$\sqrt{x^2} = |x|$

Sol(2)

$$(g \circ f)(x) = \frac{3}{x-2}$$
$$g(f(x)) = \frac{3}{x-2}$$
$$g\left(\frac{x+1}{x-2}\right) = \frac{3}{x-2}$$

Set $\boxed{y = \frac{x+1}{x-2}} \Rightarrow y(x-2) = x+1$

$$yx - 2y = x+1$$
$$yx - x = 1+2y$$
$$x(y-1) = 1+2y \Rightarrow \frac{1+2y}{y-1}$$

$\Longrightarrow$

$$g\left(\frac{x+1}{x-2}\right) = \frac{3}{x-2}$$

$$g(y) = \frac{3}{\frac{1+2y}{y-1} - 2}$$

$$= \frac{3}{\frac{1+2y}{y-1} - \frac{2(y-1)}{y-1}}$$

$$= \frac{3}{\frac{1+2y-2y+2}{y-1}}$$

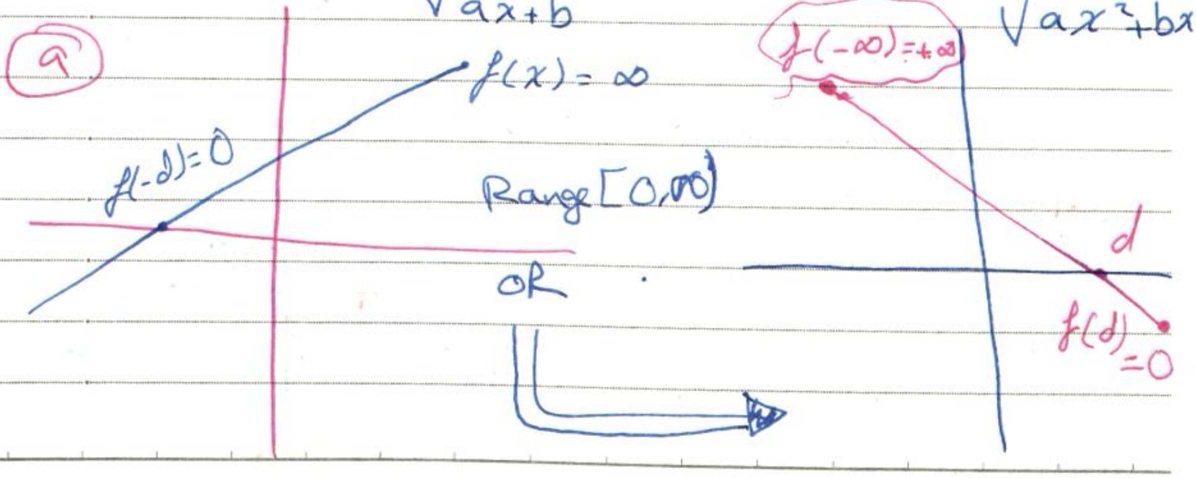
$$= \frac{3(y-1)}{3} = y-1$$

$$g(y) = y-1$$

$$\Rightarrow g(x) = x-1$$

⑫ Range of the following forms:

(a) $f(x) = \frac{c}{\sqrt{ax+b}}$ (b) $f(x) = \frac{C}{\sqrt{ax^2+bx+C}}$



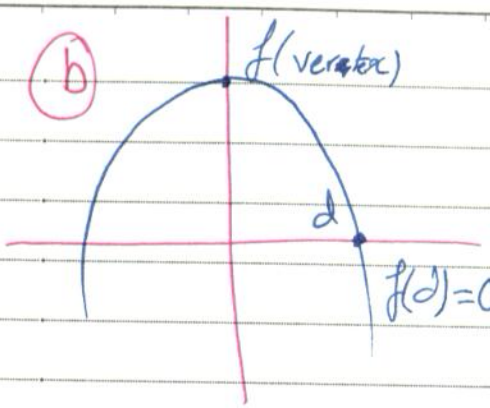
Remark

$\frac{1}{\infty} = 0$

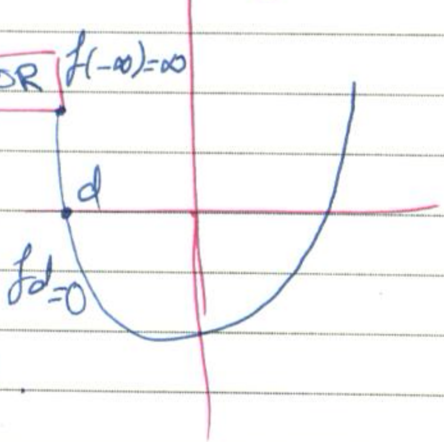
$\frac{1}{0^+} = \infty$

$\frac{1}{0^-} = -\infty$

ⓑ



$\boxed{\text{OR}}$



Ex:- Find the Range of

① $f(x) = \frac{1}{\sqrt{x-1}}$ ② $f(x) = \frac{-3}{\sqrt{4-x}}$

③ $f(x) = \frac{1}{\sqrt{x^2 - 4x}}$ ④ $f(x) = \frac{-1}{\sqrt{9+x^2}}$

Sol(1)

$f(x) = \frac{1}{\sqrt{x - 1}}$

نبحث عن القيم الموجبة للجذر

$\boxed{x - 1 \geq 0} \Rightarrow x = 1$

Domain $[1, \infty)$

- - - - 0 + +

$-\infty$ 1 ∞

$f(1) = \frac{1}{\sqrt{1-1}} = \frac{1}{0^{\oplus}} \rightarrow$ لا قيمة تحت الجذر

$= \infty$

$f(\infty) = \frac{1}{\sqrt{\infty - 1}}$

$= \frac{1}{\infty}$

$= 0$

$\Rightarrow$ Range $= (0, \infty)$

Sol

$f(x) = \frac{-3}{\sqrt{4-x}}$

$4 - x \geq 0$

+ 0 −

$-\infty$ 4 ∞

Domain $(-\infty, 4)$

$$f(-\infty) = \frac{-3}{\sqrt{4--\infty}} = \frac{-3}{\infty} = 0$$

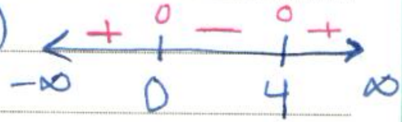
$$f(4) = \frac{-3}{\sqrt{4-4}} = \frac{-3}{0^+} = -\infty$$

$\Rightarrow$ Range $(-\infty, 0)$

Sol(3) $f(x) = \frac{1}{\sqrt{x^2-4x}}$

$x^2 - 4x \geq 0$

Domain $= (-\infty, 0) \cup (4, \infty)$



Vertex at $x = \frac{-b}{2a} = -\left(\frac{-4}{2(1)}\right) = 2$

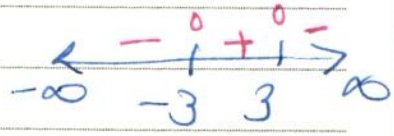
$$f(4) = \frac{1}{\sqrt{16-16}} = \frac{1}{0^+} = \infty$$

$$f(\infty) = \frac{1}{\sqrt{\infty^2 - 4\infty}} = 0$$

Range $(0, \infty)$

Sol(4) $f(x) = \frac{1}{\sqrt{9-x^2}}$

$9 - x^2 \geq 0$



Domain $(-3, 3)$

$\Longrightarrow$

$$\text{vertex} = \frac{-b}{2a} = 0$$

$$f(-3) = f(3) = \frac{-1}{\sqrt{9-9}} = \frac{-1}{0} = -\infty$$

$$f(0) = \frac{1}{\sqrt{9-0^2}} = \frac{1}{3}$$

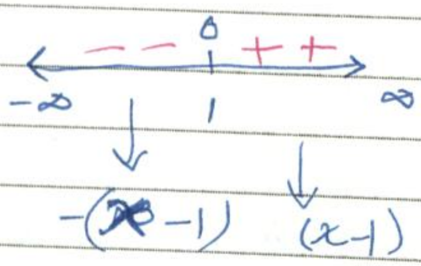
$$\text{Range} = \left(-\infty, \frac{1}{3}\right]$$

Ex: Sketch and find the domain and the Range of $f(x) = \dfrac{|x-1|}{x-1}$

Sol Re-define the absolute value function

$$|x-1|$$

$$x-1=0 \Rightarrow \boxed{x=1}$$



⟹

$$|x-1| = \begin{cases} x-1 & \text{if } x>1 \\ -(x-1) & \text{if } x<1 \\ 0 & \text{if } x=1 \end{cases}$$

OR

$$= \begin{cases} x-1 & \text{if } x\geq 1 \\ -(x-1) & \text{if } x<1 \end{cases}$$

$$f(x) = \frac{|x-1|}{x-1} = \begin{cases} \frac{x-1}{x-1} & \text{if } x>1 \\ \frac{-(x-1)}{x-1} & \text{if } x<1 \end{cases}$$

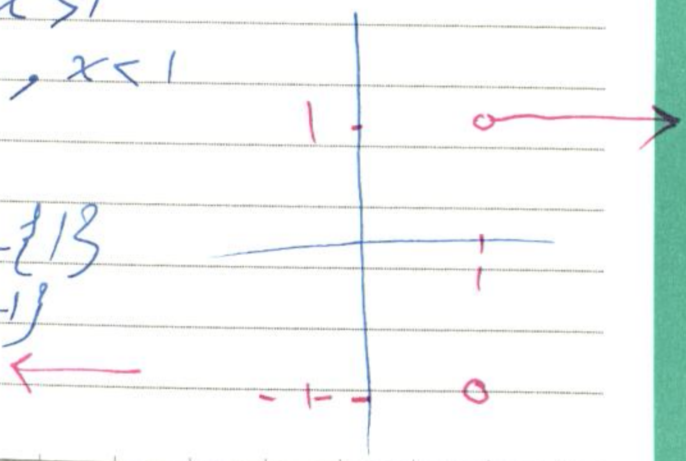
$$= \begin{cases} 1 & \text{if } x>1 \\ -1 & \text{if } x<1 \end{cases}$$

$f(x) = 1, \ x>1$

$f(x) = -1, \ x<1$

Domain $\mathbb{R} - \{1\}$

Range $\{1, -1\}$



Calculus section 4 just

25th. oct. 2016.

⑬ Limits As $x \to a^+$ or $x \to a^-$

Thm: ① $\lim\limits_{x \to 0^+} \frac{1}{x} = \frac{1}{0^+} = \infty$

نظريه ←

Theorem ② $\lim\limits_{x \to 0^-} \frac{1}{x} = \frac{1}{0^-} = -\infty$

Remark:- if $a>0$

① $a^+ - a = 0^+$

② $a^- - a = 0^-$

③ $-a^+ + a = 0^+$

④ $-a^- + a = 0^-$

$-2^+ + 2 =$

$\pm$

$1.999 + 2 = 0000...1$

$= 0^+$

Ex:- find

① $\lim\limits_{x \to 1^+} \frac{1}{x-1} = \frac{1}{1^+ - 1} = \frac{1}{0^+} = \infty$

② $\lim\limits_{x \to -2^-} \frac{1}{x+2} = \frac{1}{-2^- + 2} = \frac{1}{0^-} = -\infty$

③ $\lim\limits_{x \to -2^-} \frac{1}{x-2} = \frac{1}{-2^+ - 2} = \frac{1}{+2^+ - 2} = \frac{1}{-4}$

⑭ limits As $x \to a$

Ex:- find.

① $\lim\limits_{x \to 2} (3x^2 + 7x - 1) = 25$

② $\lim\limits_{x \to 5} \dfrac{x^2 - 25}{\sqrt{x^2+1}} = \dfrac{0}{\sqrt{26}} = 0$ ✓

③ $\lim\limits_{x \to -2} \dfrac{x^2 - 4}{x^2 + 3x - 2} = \dfrac{0}{-4} = 0$ ✓

$4+6-2$

④ $\lim\limits_{x \to 0} \dfrac{\sqrt{4+x} - 2}{x} = \dfrac{0}{0}$

⑤ $\lim\limits_{x \to 1} \dfrac{x^3 - 1}{x^2 - 1} = \dfrac{0}{0}$

⑥ $\lim\limits_{x \to 0} \dfrac{\sqrt{x+1} - 1}{\sqrt{x+4} - 2} = \dfrac{0}{0}$

Sol(4)

$$\lim_{x \to 0} \frac{\sqrt{4+x} - 2}{x} * \frac{\sqrt{4+x} + 2}{\sqrt{4+x} + 2}$$

$$= \lim_{x \to 0} \frac{\cancel{4} + \cancel{x} - \cancel{4}}{\cancel{x}(\sqrt{4+x} + 2)}$$

$$= \lim_{x \to 0} \frac{1}{\sqrt{4+x} + 2} = \frac{1}{4}$$

Sol(5) $\lim \frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{(x-1)}(x+1)} = \frac{3}{2}$

⑮ limit As $x \to -\infty$ or $x \to \infty$

Ex:- find the following

① $\lim\limits_{x \to \infty} \frac{3x^2-5x+1}{x^2+x+2}$

② $\lim\limits_{x \to \infty} \frac{x^2+1}{2x^3-x}$

③ $\lim\limits_{x \to \infty} \frac{x^3-8}{x^2-4}$

④ $\lim\limits_{x \to \infty} \frac{2x+1}{\sqrt{x^2-1}}$

⑤ $\lim\limits_{x \to -\infty} \frac{2x+3}{\sqrt{9x^2-1}}$

⑥ $\lim\limits_{x \to \infty} \frac{|x|+|x-2|}{x}$

⑦ $\lim\limits_{x \to -\infty} \frac{|x|+|x-2|}{x}$

Remark:-

① $\frac{1}{\infty} = 0$

② $\frac{1}{-\infty} = 0$

③ $|x| = x$ when $x \to \infty$

④ $|x| = -x$ when $x \to -\infty$

Sol(1) $$\lim_{x \to \infty} \frac{x^2\left(3 - \frac{5}{x} + \frac{1}{x^2}\right)}{x^2\left(1 + \frac{1}{x} + \frac{3}{x^2}\right)}$$

$$= \frac{3 - \frac{5}{\infty} + \frac{1}{(\infty)^2}}{1 + \frac{1}{\infty} + \frac{2}{(\infty)^2}} = \frac{3-0+0}{1+0+0} = \boxed{3}$$

Sol(2). $$\lim_{x \to \infty} \frac{\cancel{x^2}\left(1 + \frac{1}{x^2}\right)}{\cancel{x^3}\left(2 - \frac{1}{x^2}\right)}$$

= ~~lim~~ $$\frac{1 + \frac{1}{(\infty)^2}}{\infty\,(2 - \frac{1}{(\infty)^2}} = \frac{1}{\infty(2)} = \frac{1}{\infty} = 0$$

Sol(3). $$\lim_{x \to \infty} \frac{x^3\left(1 - \frac{8}{x^3}\right)}{x^2\left(1 - \frac{4}{x^2}\right)} = \frac{\infty\,(1 - 0)}{(1 - 0)} = \infty$$

Sol(4). $$\lim_{x \to \infty} \frac{x\left(2 + \frac{1}{x}\right)}{\sqrt{x^2\left(1 - \frac{1}{x^2}\right)}}$$

$$= \lim_{x \to \infty} \frac{x\left(2 + \frac{1}{x}\right)}{\sqrt{x^2}\sqrt{1 - \frac{1}{x^2}}}$$

$$= \lim_{x \to \infty} \frac{x\left(2 + \frac{1}{x}\right)}{|x|\sqrt{1 - \frac{1}{x^2}}}$$

$$= \lim_{x \to \infty} \frac{x\left(2 + \frac{1}{x}\right)}{x\sqrt{1 - \frac{1}{x^2}}}$$

$$= \frac{2}{\sqrt{1}} = \boxed{2}$$

Sol(5) $\lim_{x \to \underset{-}{\infty}} \dfrac{2x+3}{\sqrt{9x^2-1}}$ Malitpk.

$$= \lim_{x \to \underset{-}{\infty}} \frac{x\left(2+\frac{3}{x}\right)}{\sqrt{x^2+\left(9-\frac{1}{x^2}\right)}}$$

$$= \lim_{x \to -\infty} \frac{x\left(2+\frac{3}{x}\right)}{\sqrt{x^2}\sqrt{9-\frac{1}{x^2}}}$$

$$= \lim_{x \to -\infty} \frac{x\left(2+\frac{3}{x}\right)}{|x|\sqrt{9-\frac{1}{x^2}}}$$

$$= \lim_{x \to -\infty} \frac{\cancel{x}\left(2+\frac{3}{x}\right)}{-\cancel{x}\sqrt{9-\frac{1}{x^2}}}$$

$$= \frac{-2}{3}$$

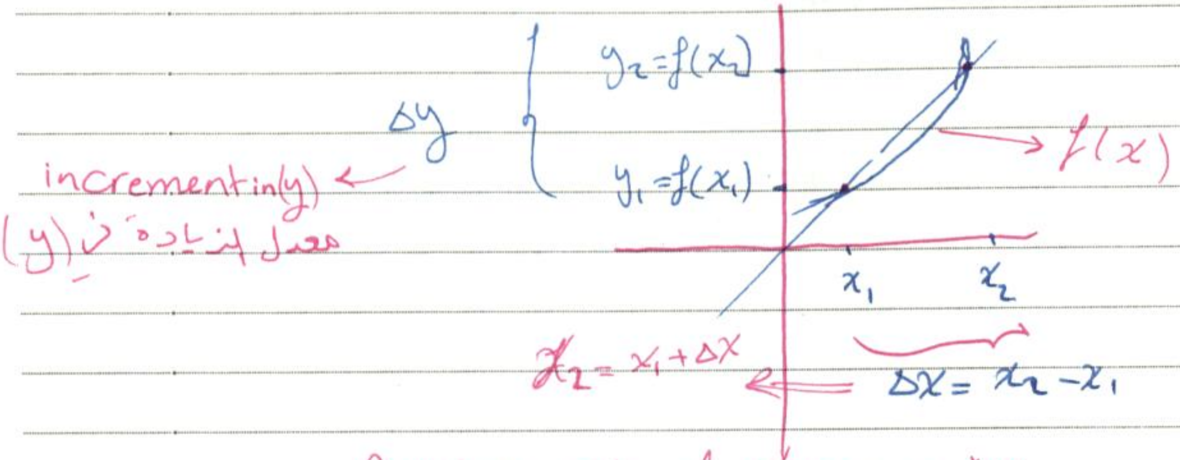
Sol(7) $\lim_{x \to -\infty} \dfrac{|x|+|x-2|}{x}$

$$= \lim_{x \to -\infty} \frac{-x+-(x-2)}{x}$$

$$= \lim_{x \to -\infty} \frac{-2x+2}{x}$$

$$= \lim_{x \to -\infty} \frac{\cancel{x}\left(-2+\frac{2}{x}\right)}{\cancel{x}(1)} = 2$$

⑯ Average rate of change معدل التغير



$$\text{Average rate of change} = \frac{\Delta y}{\Delta x}$$

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$= \frac{f(x_1 + \Delta x) - f(x_1)}{\Delta x}$$

Ex1- find Average rate of change.

① $f(x) = 3-7x$, $x=2$, $\Delta x = 0.5$

② $f(x)=\frac{3}{x}$, from $\underset{\boxed{x_1}}{\underline{x}}$ to $\underset{\boxed{x_2}}{\underline{x+\Delta x}}$

③ $f(x)=x^2$, from 2 to 7

Sol(1) Av. rate of change $= \frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{\Delta x}$

$x_2 = x_1 + \Delta x$

$= 2 + 0.5$

$= 2.5$

~~$f(x)$~~ .

$= \frac{f(2.5) - f(2)}{0.5}$

$= \frac{3-7(2.5)-(3-7(2)}{0.5}$

$= -7$.

Sol(2) Av. rate of change $= \frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{\Delta x}$

$$= \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \frac{\frac{3}{x+\Delta x} - \frac{3}{x}}{\Delta x}$$

$$= \frac{\frac{3x - 3(x+\Delta x)}{(x+\Delta x)(x)}}{\frac{\Delta x}{1}}$$

$$= \frac{\frac{\cancel{3x} - \cancel{3x} - 3\cancel{\Delta x}}{(x+\Delta x)(x)}}{\frac{\cancel{\Delta x}}{1}}$$

$$= \frac{-3}{(x+\Delta x)(x)}$$