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Department of Electrical Engineering
Second Semester 2014/2015
EE 407: Radiowave Propagation and Antennas

First Exam Sunday 22/3/2015

Q1 (7 pts.) The current distribution over a wire antenna of length L placed along the z-axis is given by:

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$$I = I_0(1-\frac{2}{L}|z'|)a_z, \qquad \frac{-L}{2} \le z' \le \frac{L}{2}$$

(a) Derive an expression for the magnetic vector potential A in the far-field. Assume $L \ll \lambda$.
(b) Derive expressions for the radiated electric and magnetic fields.
(Hint: use $\vec{E} \approx -j\omega(A_\theta a_\theta + A_\phi a_\phi)$)
(c) Derive expression for the radiation resistance of this antenna.

$$(a)\ A = \frac{\mu}{4\pi}\int \vec{I}\frac{e^{-j\beta|r-r'|}}{|r-r'|}dl' = \frac{\mu I_0 e^{-j\beta r}}{4\pi r}\int_{-L/2}^{L/2}(1-\tfrac{2}{L}z)e^{jz\cos\theta}\,dz\ a_z$$

β $\quad$ $|z|$?

$$= \frac{\mu I_0 e^{-j\beta r}}{4\pi r}\int_{-L/2}^{L/2} e^{jz\cos\theta}dz - \frac{2}{L}\int_{-L/2}^{L/2} z\, e^{jz\cos\theta}dz$$

$$\Rightarrow \frac{\mu I_0 e^{-j\beta r}\sin(\frac{L}{2}\cos\theta)}{2\pi r\cos\theta} + \frac{4}{L\cos^2\theta}\cos(\tfrac{L}{2}\cos\theta)(j\tfrac{L}{2}\cos\theta) - \frac{j4}{L\cos^2\theta}\sin(\tfrac{L}{2}\cos\theta)$$

$$A = \frac{\mu I_0 e^{-j\beta r}L}{4\pi r} + \frac{j4}{2\cos\theta}\left(\cos(\tfrac{L}{2}\cos\theta) - 1\right) a_z$$

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$$\cos(x) \cong 1,\ x \ll 1 \Rightarrow \boxed{A \cong \frac{\mu I_0 e^{-j\beta r}L}{4\pi r}a_z}$$

$$(b)\ H = \frac{1}{\mu}\vec{\nabla}\times\vec{A}, \qquad A_{spherical} = \frac{\mu I_0 L}{4\pi r}e^{-j\beta r}(\cos\theta a_r - \sin\theta a_\theta)$$

$$\vec{\nabla}\times\vec{A} = \frac{1}{r}\left[\frac{\partial(\frac{-\mu I_0 L e^{-j\beta r}}{4\pi}\sin\theta)}{\partial r} - \frac{\partial(\frac{\mu I L e^{-j\beta r}}{4\pi r}\cos\theta)}{\partial\theta}\right]a_\varphi$$

$$= \frac{1}{r}\left[\frac{+j\mu I_0 L\beta\sin\theta}{4\pi}e^{-j\beta r} + \frac{\mu I_0 L e^{-j\beta r}\sin\theta}{4\pi r}\right]a_\varphi$$

$$= \frac{\mu I_0 L\sin\theta}{4\pi}e^{-j\beta r}\left[\frac{j\beta}{r} + \frac{1}{r^2}\right]a_\varphi$$

1

$$\text{far field} \Rightarrow \boxed{\vec{H} = \frac{jI_0 L\beta\sin\theta}{4\pi r}e^{-j\beta r}a_\varphi}$$

1

Far field:-

$$\boxed{\vec{E} = \eta \vec{H} \times a_n = \frac{j30 I_o L \beta \sin\theta}{r} e^{-j\beta r} a_\theta}$$

(c) $P_{avg} = \frac{1}{2}\eta |H|^2 = 60\pi \frac{I_o^2 L^2 \beta^2 \sin^2\theta}{16\pi^2 r^2}$

$$= \frac{60}{16\pi}\left(\frac{I_o L \beta \sin\theta}{r}\right)^2 a_r$$

$$P_{rad} = \iint \frac{60 I_o^2 L^2 \beta^2 \sin^2\theta}{16\pi r^2} a_r \, r^2 \sin\theta \, d\theta \, d\varphi \, a_r$$

$$= \frac{60 I_o^2 L^2 \beta^2}{16\pi} \int_0^{2\pi}\int_0^{\pi} \sin^3\theta \, d\theta \, d\varphi$$

$$= \frac{120 I_o^2 L^2 \beta^2}{16} \int_0^{\pi} \sin^3\theta \, d\theta = 10 I_o^2 L^2 \beta^2 \text{ W}$$

$\cancel{P_{rad} = \frac{1}{2} I_o^2 R_{rad}}$ $P_{rad} = \frac{1}{2} I_o^2 R_{rad}$ 2

$$\Rightarrow \boxed{R_{rad} = \frac{2P_{rad}}{I_o^2} = 20 L^2 \beta^2 \ \Omega}$$

Q2 (10 pts.) An antenna in free space has a far-zone E-field given by: $E = 24\frac{\sin(2\theta)}{r}e^{-j\beta r}a_\theta$ نور ماجد خضير

Determine the following: 9/10

(a) H.
(b) Radiation intensity $U(\theta,\phi)$.
(c) The radiated power P_{rad}.
(d) The directivity D.
(e) The HPBW and FNBW.
(f) Plot the radiation pattern.

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(a) $\vec{H} = \frac{E_\theta}{\eta_o} a_\varphi = \cancel{\quad} = \boxed{\frac{\sin(2\theta)}{5\pi r} e^{-j\beta r} a_\varphi}$

(b) $P_{avg} = \frac{|E|^2}{2\eta} = \frac{576\sin^2(2\theta)}{240\pi r^2} a_r$

$\boxed{U(\theta,\varphi) = r^2 P_{avg} = \frac{12\sin^2(2\theta)}{5\pi} a_r}$ 1

(c) $P_{rad} = \int_0^{2\pi}\int_0^{\pi} U(\theta,\varphi)\sin\theta\, d\theta\, d\varphi$

$\cancel{\quad} = \frac{24}{5}\int_0^{\pi}\sin^2(2\theta)\cdot\sin\theta\, d\theta$

$\Rightarrow \boxed{P_{rad} = 5.12 \text{ W}}$ 3

(d) $D = \frac{4\pi U_{max}}{P_{rad}}$, $U_{max} = \frac{12}{5\pi}$

$\Rightarrow \boxed{D = \frac{4\pi \times 12}{5\pi \times 5.12} = 1.875}$ 2

(e) $D = G_{d,max}$

$HPBW = \frac{1}{2}G_{dmax} = \boxed{0.9375}$ $\Rightarrow G_d = \frac{4\pi U(\theta,\varphi)}{P_{rad}}$

$\theta_2 = ?$

~~FNBW~~ $\Rightarrow 0.9375 = \frac{4\pi}{5.12}\cdot\frac{12\sin^2(2\theta)}{5\pi}$ $\Rightarrow \sin^2(2\theta) = 0.5$

$\Rightarrow \sin(2\theta) = \frac{\sqrt{2}}{2}$ $\Rightarrow \theta_1 = \frac{\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)}{2} = \boxed{\frac{\pi}{8}}$

Q3 (8 pts.) Two identical small current loops with radius **a** placed a distance **d** apart are fed with same current amplitude I_0 in phase ($\alpha = 0$), according to the figure below.

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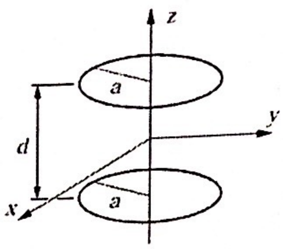
(a) Calculate the array factor for this array.
(b) Determine the array pattern of this array.
(c) Determine the smallest separation **d** so that nulls in the array pattern are formed in the directions $\theta = 0^\circ, 60^\circ, 120^\circ$, and 180°, where θ is the angle measured from the positive z axis.



a) $AF = \dfrac{\sin\left(\frac{N\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)}$, $\psi = \beta d\cos\theta + \alpha$

$= \beta d \cos\theta$

2

$\Rightarrow AF = \dfrac{\sin(\beta d\cos\theta)}{\sin\left(\frac{\beta d}{2}\cos\theta\right)}$

b) $f(\theta) = |\sin\theta|$

2

Array Pattern $= |\sin\theta| \cdot \dfrac{\sin(\beta d\cos\theta)}{\sin\left(\frac{\beta d}{2}\cos\theta\right)}$

c) Nulls $\Rightarrow \psi = \frac{\pi}{2}$?

$\Rightarrow \frac{\pi}{2} = \beta d\cos\theta$

@ $\theta = 0 \Rightarrow d = \frac{\pi}{2\beta}$

@ $\theta = 60^\circ \Rightarrow d = \frac{\pi}{\beta}$?

@ $\theta = 120^\circ \Rightarrow d = \frac{-\pi}{\beta}$

@ $\theta = 180^\circ \Rightarrow d = \frac{-\pi}{2\beta}$