# التكامل بالتعويض جزء ٢

تدريب (٣)

(٣) جد $\int_{+1}^{-1} (4س - 1)\sqrt[3]{س - 2س^2 - 1}\, دس$

الحل: (١) $ص = س - 2س^2 - 1$

$س = 1 \Leftarrow ص = 1 - 2(1)^2 - 1 \Leftarrow ص = 1 - 2 - 1 = -2$

$س = -1 \Leftarrow ص = -1 - 2(-1)^2 - 1 \Leftarrow ص = -1 - 2 - 1 = -4$

(٢) $دص = (1 - 4س)\, دس$

(٣) $دس = \frac{دص}{1 - 4س}$

(٤) $-\int_{-2}^{-4} \cancel{(4س - 1)}\sqrt[3]{ص}\, \frac{دص}{\cancel{1 - 4س}}$

$-\int_{-2}^{-4} ص^{\frac{1}{3}}\, دص = -\frac{3ص^{\frac{4}{3}}}{4}\Big]_{-2}^{-4}$

$-\frac{3}{4}\left[\sqrt[3]{ص^4}\right]_{-2}^{-4} = -\frac{3}{4}\left[\sqrt[3]{(-4)^4} - \sqrt[3]{(-2)^4}\right]$

$= -\frac{3}{4}\left[\sqrt[3]{256} - \sqrt[3]{16}\right]$ #

④ $\int_{0}^{2} \frac{1}{\sqrt[5]{1+س}}\, دس$

① $ص = س + 1$

$دص = دس$

$س = 0 \Leftarrow ص = 0 + 1 = 1$

$س = 2 \Leftarrow ص = 2 + 1 = 3$

$\int_{1}^{3} \frac{1}{\sqrt[5]{ص}}\, دص$

$\int_{1}^{3} \left(\frac{1}{ص^{\frac{1}{5}}}\right) دص$

$\int_{1}^{3} ص^{-\frac{1}{5}}\, دص$

$= 5ص^{\frac{4}{5}} \Big]_{1}^{3}$

$\frac{5}{4}\left[\sqrt[5]{ص^4}\right]_{1}^{3}$

$\frac{5}{4}\left[\sqrt[5]{(3)^4} - \sqrt[5]{1^4}\right]$

قاعده مهمه جداً

وحديه

$\int (أس + ب)^{ن}\, دس$

$\frac{(أس + ب)^{ن+1}}{أ \times (ن+1)} + جـ$

حقائق

$\int_{0}^{2} \frac{1}{\sqrt[5]{1+س}}\, دس$

$\int_{0}^{2} \left(\frac{1}{(س+1)^{\frac{1}{5}}}\right) دس$

$\int_{0}^{2} (س+1)^{-\frac{1}{5}}\, دس$

$= 5\frac{(س+1)^{\frac{4}{5}}}{4} \Big]_{0}^{2}$

$\frac{5}{4}\left[(3)^{\frac{4}{5}} - (1)^{\frac{4}{5}}\right]$

تدريب ٤ جد قيمة $\int (\text{أ س} + \text{ب})^{\text{ن}} \, \text{د س}$ ، أ ≠ ٠ ، ن ≠ -١

⇐ قاعدة مهمة وسهلة وحرجة

$$\int (\text{أ س} + \text{ب})^{\text{ن}} \, \text{د س}$$

اقتران خطي س بدون أس

$$= \frac{(\text{أ س} + \text{ب})^{\text{ن}+١}}{\text{أ} \times (\text{ن} + ١)} + \text{جـ}$$

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مثال: $\int (\text{٥ س} + ٤)^{٧} \, \text{د س}$

$$= \frac{(\text{٥ س} + ٤)^{٨}}{٨ \times ٥} + \text{جـ}$$

$$= \frac{(\text{٥ س} + ٤)^{٨}}{٤٠} + \text{جـ}$$

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مثال: $\int \sqrt{٤ - ٣\text{س}} \, \text{د س} = \int (٤ - ٣\text{س})^{\frac{١}{٢}} \, \text{د س}$

$$= \int (٤ - ٣\text{س})^{\frac{١}{٢}} \, \text{د س} = \frac{(٤ - ٣\text{س})^{\frac{٣}{٢}}}{\cancel{٣} \times -٤} + \text{جـ}$$

$$= \frac{-(٤ - ٣\text{س})^{\frac{٣}{٢}}}{٤} + \text{جـ}$$

مثال : $\int \text{جا}(\underbrace{٥س - ٣}_{\text{زاوية}})\, \text{دس}$

$\text{ص} = ٥س - ٣$

$\text{دص} = ٥\,\text{دس} \Leftarrow \text{دس} = \frac{\text{دص}}{٥}$

$\int \text{جا ص} \frac{\text{دص}}{٥} = \frac{١}{٥} \int \text{جا ص دص}$

$= -\frac{١}{٥}\, \text{جتا ص} + \text{جـ}$

$$\boxed{= -\frac{١}{٥}\, \text{جتا}(٥س - ٣) + \text{جـ}}$$

$\int \text{جا}(\underbrace{\text{أس} + \text{ب}}_{\text{خطية}})\, \text{دس} = \frac{-١}{\text{أ}}\, \text{جتا}(\text{أس} + \text{ب}) + \text{جـ}$

معامل

$\int \text{جتا}(\underbrace{\text{أس} + \text{ب}}_{\text{خطية}})\, \text{دس} = \frac{١}{\text{أ}}\, \text{جا}(\text{أس} + \text{ب}) + \text{جـ}$

$\int \text{جا}(\underline{٥س - ٣})\, \text{دس}$

$$\boxed{= -\frac{١}{٥}\, \text{جتا}(٥س - ٣) + \text{جـ}}$$

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$$\int \text{قا}(٣ - ٥\text{س})\, \text{دس} = \frac{١}{-٥}\, \text{ظا}(٣ - ٥\text{س}) + \text{جـ}$$

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## * قاعده

① $$\int (\text{أس} + \text{ب})^{\text{ن}}\, \text{دس} = \frac{(\text{أس} + \text{ب})^{\text{ن}+١}}{\text{أ}(\text{ن}+١)} + \text{جـ}$$

② $$\int \text{جا}(\text{أس} + \text{ب})\, \text{دس} = -\frac{\text{جتا}(\text{أس} + \text{ب})}{\text{أ}} + \text{جـ}$$

③ $$\int \text{جتا}(\text{أس} + \text{ب})\, \text{دس} = \frac{\text{جا}(\text{أس} + \text{ب})}{\text{أ}} + \text{جـ}$$

④ $$\int \text{قا}^{٢}(\text{أس} + \text{ب})\, \text{دس} = \frac{\text{ظا}(\text{أس} + \text{ب})}{\text{أ}} + \text{جـ}$$

مثال ٣: جد $\int_{0}^{1} \frac{1}{\sqrt[3]{2س-1}}\, دس$

$$\int_{0}^{1} \left(\frac{1}{(2س-1)^{\frac{1}{3}}}\right) دس = \int_{0}^{1} (2س-1)^{-\frac{1}{3}}\, دس$$

$$\left[\frac{3(2س-1)^{\frac{2}{3}}}{2\times 2}\right]_{0}^{1} = \left[\frac{3}{4}\sqrt[3]{(2س-1)^2}\right]_{0}^{1}$$

$$\left[\frac{3}{4}\sqrt[3]{(2\times 0-1)^2} - \frac{3}{4}\sqrt[3]{(2\times 1-1)^2}\right]$$

$$\left[\frac{3}{4}\sqrt[3]{(-1)^2} - \frac{3}{4}\sqrt[3]{(27-1)^2}\right]$$

$$\left[\frac{3}{4}\sqrt[3]{1} - \frac{3}{4}\sqrt[3]{27\times 27}\right]$$

$$\left[\frac{3}{4}\left(1 - 3\times 3\times 3\times 3\times 3\times 3\right)\right]$$

$$\left[\frac{3}{4}\left(1 - 3\times 3\right)\right] = \frac{3}{2}\times 8 = \frac{24}{4}$$

$$= 6$$

# التكامل بالتعويض جـ٣

**تدريب ٥** ① جد $\int_{-1}^{2} ٦(١-٢س)^{٥}\, دس$

$$\int (أس+ب)^{ن}\, دس = \frac{(أس+ب)^{ن+١}}{(ن+١)\,أ} + جـ$$

$$\left[\frac{٦(١-٢س)^{٦}}{-٢}\right]_{-١}^{٢} = (١-٢\times٢)^{٦} - (١-٢\times-١)^{٦}$$

$$= (١-٤)^{٦} - (١+٢)^{٦}$$

$$(-٣)^{٦} - (٣)^{٦} = ٣^{٦} - ٣^{٦} = ٠ \text{ صفر}$$

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② $\int ١٢\, جا(١-٤س)\, دس$

$$\frac{-١٢\, جتا(١-٤س)}{-٤} + جـ$$

$$٣\, جتا(١-٤س) + جـ$$

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* قاعدة $\int جا(أس+ب)\, دس$

$$= \frac{-١}{أ}\, جتا(أس+ب)$$

$$= \frac{-جتا(أس+ب)}{أ} + جـ$$

## الاسئلة (١)

أ) $\int (1-2س)(س - س^2)^4 \, دس$

الحل: $ص = س - س^2 \, دس$

$دص = (1-2س) \, دس$

$دس = \frac{دص}{1-2س}$

$\int \cancel{(1-2س)} \, ص^4 \, \frac{دص}{\cancel{1-2س}} = \frac{ص^5}{5} + ج$

$\frac{(س - س^2)^5}{5} + ج$

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ب) $\int 6س^5 \sqrt{(2س^3-2)} \, دس$

$ص = 2س^3 - 2$

$دص = 6س^2 \, دس$

$دس = \frac{دص}{6س^2}$

$\int \cancel{6س^2} \, ص^{\frac{1}{2}} \, \frac{دص}{\cancel{6س^2}} = \frac{2ص^{\frac{3}{2}}}{3} + ج$

$(2س^3-2)^{\frac{3}{2}} + ج$

(ج)

$$\int \frac{(2س - 3س^2)}{\text{زاوية}} \sqrt{(س^3 - س^2)}\, دس$$

$$ص = س^3 - س^2 \Leftarrow \frac{دص}{\ } = (3س^2 - 2س)\, دس$$

$$دس = \frac{دص}{3س^2 - 2س}$$

$$\int \frac{\cancel{(2س - 3س^2)} \sqrt{ص}\, دص}{\cancel{3س^2 - 2س}}$$

$$- \int \sqrt{ص}\, دص$$

$$= - \frac{1}{\frac{1}{2}} ص^{\frac{1}{2}} + جـ$$

$$= - \frac{1}{\frac{1}{2}} (س^3 - س^2)^{\frac{1}{2}} + جـ$$

$$\frac{ص - س}{ص - س} = -1$$

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(د)

$$\int \frac{3س - 9}{(س^2 - 6س)^2}\, دس$$

$$ص = س^2 - 6س$$

$$دص = (2س - 6)\, دس$$

$$\frac{دص}{2س - 6} = دس$$

$$\int \frac{\cancel{(س - 3)}\, 3}{ص^2} \cdot \frac{دص}{\cancel{(س - 3)}\, 2}$$

$$\int ص^{-2}\, دص \quad \frac{3}{2}$$

$$\frac{3}{2} \frac{ص^{-1}}{-1} + جـ$$

$$\frac{-3}{2ص} + جـ$$

$$= \frac{-3}{2(س^2 - 6س)} + جـ$$

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② $\int \sqrt[3]{(3س-2)^2}\, دس$

$\int (3س-2)^{\frac{2}{3}}\, دس$

$\frac{(3س-2)^{\frac{5}{3}}}{5} + ج$

$\int (أس+ب)^{ن}\, دس = \frac{(أس+ب)^{ن+1}}{(ن+1) \times أ} + ج$

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③ $\int (س-1)(2س^2-4س+1)^5\, دس$

$ص = 2س^2-4س+1$

$دص = (4س-4)\, دس$

$دس = \frac{دص}{4س-4}$

$دس = \frac{دص}{4(س-1)}$

$\int \cancel{(س-1)}\, (ص)^5 \frac{دص}{\cancel{(س-1)}\, 4}$

$\frac{1}{4} \frac{ص^6}{6} + ج = \frac{(2س^2-4س+1)^6}{24} + ج$

ج) $\int \text{جتا}^2(٢-س)\,دس$

$= \dfrac{٢\,\text{جا}(٢-س)}{-١} + جـ$

$= -٢\,\text{جا}(٢-س) + جـ$

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د) $\int ٢س^3\,\text{جا}(س^4+١)\,دس$

$ص = س^4 + ١$

$دص = ٤س^3\,دس \Leftarrow دس = \dfrac{دص}{٤س^3}$

$\int \cancel{٢س^3}\,\text{جا}\,ص\,\dfrac{دص}{\cancel{٤س^3}}$

$\frac{١}{٢}$

$-\frac{١}{٢}\,\text{جتا}\,ص + جـ$

$-\frac{١}{٢}\,\text{جتا}(س^4+١) + جـ$

(ب) $\int_{-1}^{1} ٣س^{٢}(س^{٣}-١)^{٣}\,دس$ = صفر

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(ج) $\int_{٠}^{١} س\sqrt[٣]{س^{٢}+١}\,دس$

$ص = س^{٢} - ١$

$دص = ٢س\,دس$

$دس = \frac{دص}{٢س}$

$\int_{-١}^{٠} \cancel{٢س}\sqrt[٣]{ص}\,\frac{دص}{\cancel{٢س}}$

$\int_{٠}^{١} ص^{\frac{١}{٣}}\,دص = \left[\frac{٣ص^{\frac{٤}{٣}}}{٤}\right]_{٠}^{١}$

$\frac{٣}{٤}\left[\sqrt[٣]{ص^{٤}}\right]_{٠}^{١}$

$\frac{٣}{٤}\left[\sqrt[٣]{(١)^{٤}} - \sqrt[٣]{٠^{٤}}\right]$

$\frac{٣}{٤}\left[١ - ٠\right] = \frac{٣}{٤}$

(أ) $\int_{-١}^{١}$

$ص = ١ - س^{٢}$

$س = ٠ \Leftarrow ص = ١ - ٠^{٢} = ١$

$س = ١ \Leftarrow ص = ١ - ١^{٢} = ٠$

ج) أ) $\int_{0}^{2} \sqrt{(4س+1)}\, دس$

$= \int_{0}^{2} (4س+1)^{\frac{1}{2}}\, دس$

$= \left[ \frac{2(4س+1)^{\frac{3}{2}}}{3} \right]_{0}^{2}$

$\frac{2}{3} \left[ (9)^{\frac{3}{2}} - (1)^{\frac{3}{2}} \right]$

$\frac{2}{3} \left[ \sqrt{9^3} - 1 \right]$

$\frac{2}{3} \left[ 9 \times 3 - 1 \right]$

$\frac{2}{3} \left[ 26 \right] = \frac{52}{3}$

⑤ $\int_{1}^{2} \frac{3 - 2س}{(س^2 - 3 - 2س)^2}\, دس$

ص = س² − ٣ − ٢س

دص = (٢س − ٢) دس

دس = $\frac{دص}{٢س - ٢}$

س = ١ ⟸ ص = ١ − ٢ − ٣ = −٤

س = ٢ ⟸ ص = ٤ − ٣ − ٢ × ٢ = ٤ − ٦ = −٢

$\int_{-4}^{-2} \frac{1}{ص^2}\, دص$

= $\frac{-1}{ص}$ ]$_{-4}^{-2}$

= $\frac{-1}{-2} - \frac{-1}{-4}$

= $\frac{1}{2} - \frac{1}{4}$

= $\frac{1}{4}$

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④ ق(−٨) = ٥ ، ق(٢٧) = −٦ جد قيمة

$\int_{-8}^{27} ٣س^2\, ق'(س^3)\, دس$

ص = س³ ⟸ دص = ٣س² دس ⟸ دس = $\frac{دص}{٣س^2}$

س = −٢ ⟸ ص = (−٢)³ = −٨

س = ٣ ⟸ ص = ٣³ = ٢٧

$\int_{-8}^{27} \cancel{٣س^2}\, ق'(ص)\, \frac{دص}{\cancel{٣س^2}}$ = [ق(ص)]$_{-8}^{27}$

= ق(٢٧) − ق(−٨)

= −٦ − ٥ = −١١

⑤ .

$$\int_{0}^{2} \text{ق}(\text{س})\,\text{دس} = 3 \quad \text{حدد قيمة التكامل}$$

$$\int_{-1}^{2} \text{ق}(\text{س}^2+1)\,\text{دس}$$

$$\text{ص} = \text{س}^2 + 1$$

$$\text{دص} = 2\text{س}\,\text{دس} \Leftarrow \text{دس} = \frac{\text{دص}}{2\text{س}}$$

$$\text{س} = -1 \Leftarrow \text{ص} = (-1)^2 + 1 = 1 + 1 = 2$$

$$\text{س} = 2 \Leftarrow \text{ص} = (2)^2 + 1 = 4 + 1 = 5$$

$$\int_{2}^{5} \text{ق}(\text{ص}) \frac{\text{دص}}{2\cancel{\text{س}}}$$

$$\frac{1}{2}\int_{2}^{5} \text{ق}(\text{ص})\,\text{دص}$$

$$\frac{1}{2} \times 3 = 1.5$$